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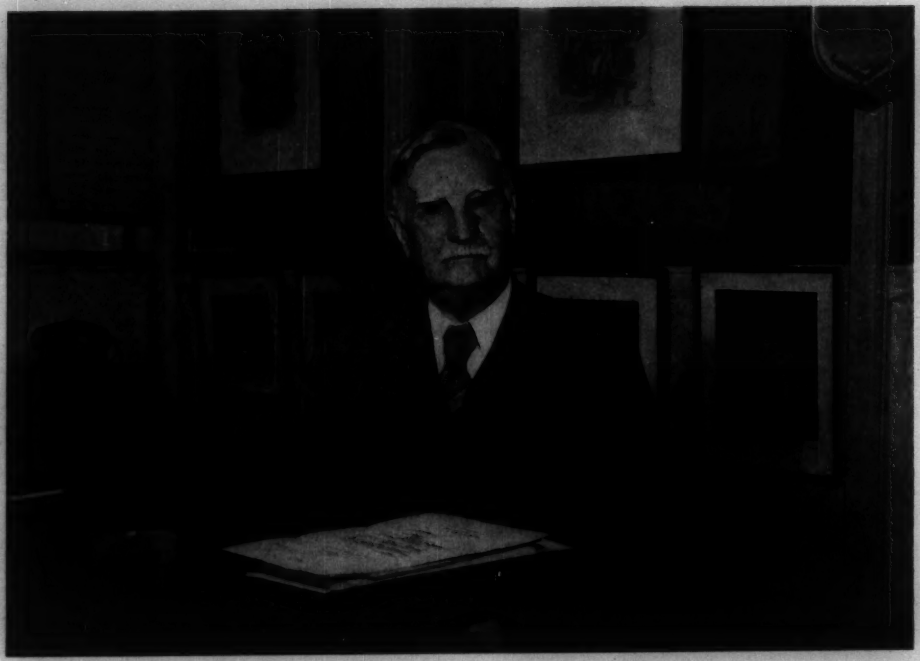

EDWARD GOWEN BUDD

Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 243 MARCH, 1947 No. 3

EDWARD GOWEN BUDD

1870–1946

On November 30, 1946, Edward Gowen Budd, a distinguished member of our Board, passed away. His spirit and influence represent the combination of a man of high moral character with that of a great pioneer.

Mr. Budd's contribution to industry is noteworthy. In the automotive field he developed the All Steel Body, the Automobile Steel Wheel, and the Stainless Steel Truck Trailer. In the railway field he developed the Stainless Steel lightweight passenger trains. Perhaps not so well known were his many contributions in the development of fabrication methods in steel stampings and productions welding. He instigated the development of the well known *shotweld* Process for Stainless Steel Fabrication.

Mr. Budd has built up a large industry. The Budd Company is well known as a major manufacturer in the Automotive field and it also ranks as a principal railway car manufacturer. Nearly every automobile embodies the All Steel Body, a basic Budd contribution; the present modern high speed streamlined train, in a very real sense, originated from the Pioneer *zephyr*.

His character as a great American citizen was perhaps best shown by his unique leadership out of the depression in the early thirties. He pioneered under great difficulties and untold opposition in the development of a new field, culminating in the famous Pioneer *zephyr*. The most depressed industry of all—the Railroads—rebounded to life, reacting with optimism, and in turn stimulated the revival of industry throughout the country. Mr. Budd set the pace and pointed to new frontiers in American industry. He had faith in our American system and the future.

Mr. Budd was a typical American individualist. He influenced

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

many men to believe in honest and useful work, along with independent thinking, as a creed for living. There were no barriers or military setups in contacting Mr. Budd. The humblest worker always had access to his personal consideration.

It is quite befitting to note that Mr. Budd was a true friend of Franklin. He was grateful for an early education at The Franklin Institute. He participated in many of our basic activities. He has been a long time member of our Board of Managers. He was interested in our Library. He was on the Special Committee and was particularly helpful in advice in the new Franklin Institute Laboratories for Research and Development.

The most touching phase of Mr. Budd's life was his simplicity and sincerity among his friends and his genuine interest in their health and progress. But a crowning achievement was a simple and beautiful home life.

Any association with Mr. Budd soon disclosed a serene partnership with a great lady, Mrs. Edward Gowen Budd, who in no small measure contributed to the successes of his life.

THEREFORE, BE IT RESOLVED, that this statement regarding Mr. Budd's activities and influence be spread upon our minutes, and be it further

RESOLVED that a copy be duly prepared and forwarded to the family of Mr. Budd with sincere expressions of regret and sympathy from the BOARD OF MANAGERS OF THE FRANKLIN INSTITUTE.

President
Secretary

January Fifteenth
Nineteen Hundred and Forty-seven

ON THE SO-CALLED PRINCIPLE OF LEAST WORK METHOD.

BY

M. Z. KRZYWOBLOCKI,

Dr. Aero. E., Associate Professor of Aero. Engr., University of Illinois.

INTRODUCTION.

Certain failures of the procedure based upon the Castigliano theorem of least work or the so-called least work method originated the following remarks. These failures occurred in some straightforward applications of the theorem in the calculation of stresses in thin-walled structures. The usual presentation of the least work method may lead to approximate results lying too far from actual conditions. Consequently the results cannot be reliable. The author proposes a different presentation of this method which follows: When applying Castigliano's theorem of least work to specific problems, we must be assured a priori that the redundant generalized forces or unknown stress distributions are expressed by functions among which there is one and only one function which gives a smaller value to the total strain-energy than any other gives having first taken the given values on the boundary. Being assured that such a minimum exists, we present the total strain-energy as a function of the redundant generalized forces or unknown coefficients and next we express in an analytical form the statement that the total strain-energy has a stationary value for small variations in the redundant forces, by equating to zero the first partial derivatives of the strain-energy with respect to each of the redundant forces or unknown coefficients. The last equations give us the values of the redundant forces or of the unknown coefficients.

1. Let $u = F(x, y)$ be the explicit function of two independent variables x and y. Deeper considerations on the necessary and sufficient conditions of the existence of a minimum of the function u will be omitted here and the reader may refer to special books on this subject [1, 2, 3, 8]. From a general standpoint, it usually is sufficient to represent the necessary and sufficient conditions that $F(x_0, y_0)$ is a minimum, for $x = x_0$, and $y = y_0$, in the form:

$$\text{(a)} \quad \left(\frac{\partial F}{\partial x}\right) = 0, \quad \left(\frac{\partial F}{\partial y}\right) = 0, \text{ which give: } x = x_0,\ y = y_0,$$

$$\text{(b)} \quad \left(\frac{\partial^2 F}{\partial x_0^2}\right) > 0 \quad \text{and} \quad \left(\frac{\partial^2 F}{\partial y_0^2}\right) > 0 \text{ (must be positive)}, \qquad (1)$$

$$\text{(c)} \quad \left(\frac{\partial^2 F}{\partial x_0 \partial y_0}\right)^2 - \left(\frac{\partial^2 F}{\partial x_0^2}\right)\left(\frac{\partial^2 F}{\partial y_0^2}\right) < 0 \text{ (must be negative)},$$

or

$$\begin{vmatrix} \left(\frac{\partial^2 F}{\partial x_0^2}\right) & \left(\frac{\partial^2 F}{\partial x_0 \partial y_0}\right) \\ \left(\frac{\partial^2 F}{\partial y_0 \partial x_0}\right) & \left(\frac{\partial^2 F}{\partial y_0^2}\right) \end{vmatrix} > 0. \tag{1a}$$

The conditions (b) and (c) may be presented in another form, namely that the matrix

$$\begin{vmatrix} \left(\frac{\partial^2 F}{\partial x_0^2}\right) & \left(\frac{\partial^2 F}{\partial x_0 \partial y_0}\right) \\ \left(\frac{\partial^2 F}{\partial y_0 \partial x_0}\right) & \left(\frac{\partial^2 F}{\partial y_0^2}\right) \end{vmatrix} > 0 \tag{1b}$$

must be definite positive. This means that both the determinants must be positive:

$$\left|\left(\frac{\partial^2 F}{\partial x_0^2}\right)\right| > 0, \qquad \begin{vmatrix} \left(\frac{\partial^2 F}{\partial x_0^2}\right) & \left(\frac{\partial^2 F}{\partial x_0 \partial y_0}\right) \\ \left(\frac{\partial^2 F}{\partial y_0 \partial x_0}\right) & \left(\frac{\partial^2 F}{\partial y_0^2}\right) \end{vmatrix} > 0. \tag{1c}$$

In the case in which $\left(\frac{\partial^2 F}{\partial x_0 \partial y_0}\right)^2 - \left(\frac{\partial^2 F}{\partial x_0^2}\right)\left(\frac{\partial^2 F}{\partial y_0^2}\right)$ is positive, it may happen that one of the partial singular values is a maximum and the other is a minimum (a saddle); and so the conditions requisite for a total minimum would not be fulfilled. If $\left(\frac{\partial^2 F}{\partial x_0 \partial y_0}\right)^2 = \left(\frac{\partial^2 F}{\partial x_0^2}\right)\left(\frac{\partial^2 F}{\partial y_0^2}\right)$, then this case is an ambiguous one and requires a special consideration, which may be found in the given references. One of the conclusions which may be reached in this case is that either $\left(\frac{\partial F}{\partial x}\right)$ or $\left(\frac{\partial F}{\partial y}\right)$ undergo no variation. Consequently there may be a partial minimum with respect to one of the variables, while the other may be such that corresponding to its variations the function is constant. Hence one may have a locus of such partial minima.

If the conditions (1) are not fulfilled it may be extremely difficult to obtain a minimum.

2. Let $u = F(x, y, z)$ be an explicit function of three independent variables x, y, and z. Again the discussion of the necessary and sufficient conditions of the existence of a minimum of the function u will be omitted. One of the usual forms of representing the necessary and sufficient conditions that $F(x_0, y_0, z_0)$ is a minimum for $x = x_0$, $y = y_0$,

$z = z_0$ is [4]:

$$\text{(a)}\quad \left(\frac{\partial F}{\partial x}\right) = 0, \quad \left(\frac{\partial F}{\partial y}\right) = 0, \quad \left(\frac{\partial F}{\partial z}\right) = 0,$$

$$x = x_0, \quad y = y_0, \quad z = z_0,$$

$$\text{(b)}\quad \left(\frac{\partial^2 F}{\partial x_0^2}\right) > 0, \quad \left(\frac{\partial^2 F}{\partial y_0^2}\right) > 0, \quad \left(\frac{\partial^2 F}{\partial z_0^2}\right) > 0,$$

$$\text{(c)}\quad \left(\frac{\partial^2 F}{\partial x_0^2}\right)\left(\frac{\partial^2 F}{\partial y_0^2}\right) - \left(\frac{\partial^2 F}{\partial x_0 \partial y_0}\right)^2 > 0,$$

$$\left(\frac{\partial^2 F}{\partial x_0^2}\right)\left(\frac{\partial^2 F}{\partial z_0^2}\right) - \left(\frac{\partial^2 F}{\partial y_0 \partial z_0}\right)^2 > 0, \tag{2}$$

$$\left(\frac{\partial^2 F}{\partial y_0^2}\right)\left(\frac{\partial^2 F}{\partial z_0^2}\right) - \left(\frac{\partial^2 F}{\partial y_0 \partial z_0}\right)^2 > 0.$$

The conditions (b) and (c) may be given in another form, namely that the matrix be definite positive:

$$\begin{vmatrix} \left(\frac{\partial^2 F}{\partial x_0^2}\right) & \left(\frac{\partial^2 F}{\partial x_0 \partial y_0}\right) & \left(\frac{\partial^2 F}{\partial x_0 \partial z_0}\right) \\ \left(\frac{\partial^2 F}{\partial y_0 \partial x_0}\right) & \left(\frac{\partial^2 F}{\partial y_0^2}\right) & \left(\frac{\partial^2 F}{\partial y_0 \partial z_0}\right) \\ \left(\frac{\partial^2 F}{\partial z_0 \partial x_0}\right) & \left(\frac{\partial^2 F}{\partial z_0 \partial y_0}\right) & \left(\frac{\partial^2 F}{\partial z_0^2}\right) \end{vmatrix} > 0. \tag{2a}$$

This means that all three determinants marked by dotted lines must be simultaneously positive.

3. Let $u = F(x_1, x_2, x_3, \cdots x_n)$ be a function of n independent variables. Similarly as in the previous chapters, the necessary and sufficient conditions that u is a minimum when $x_1 = x_{10}, x_2 = x_{20}, \cdots x_n = x_{n0}$ are:

$$\text{(a)}\quad \left(\frac{\partial F}{\partial x_1}\right) = 0, \quad \left(\frac{\partial F}{\partial x_2}\right) = 0, \quad \cdots \left(\frac{\partial F}{\partial x_n}\right) = 0,$$

which gives: $x_1 = x_{10}, \quad x_2 = x_{20}, \quad \cdots x_n = x_{n0},$

(b) the quadratic form

$$\sum_{i=1}^{n}\sum_{k=1}^{n} A_{ik}\xi_i\xi_k > 0,$$

where

$$A_{ik} = \left(\frac{\partial^2 F}{\partial x_{i0}\partial x_{k0}}\right) \tag{3}$$

must be a definite positive for all the possible systems of values ξ_ν, except when these quantities are all simultaneously zero [5, 8]. The necessary and sufficient condition for a quadratic form to be a definite positive is that the matrix must be definite positive:

$$\text{(b)} \quad \begin{vmatrix} \left(\frac{\partial^2 F}{\partial x_{10}^2}\right) & \left(\frac{\partial^2 F}{\partial x_{10}\partial x_{20}}\right) & \left(\frac{\partial^2 F}{\partial x_{10}\partial x_{30}}\right) & \cdots & \left(\frac{\partial^2 F}{\partial x_{10}\partial x_{n0}}\right) \\ \left(\frac{\partial^2 F}{\partial x_{20}\partial x_{10}}\right) & \left(\frac{\partial^2 F}{\partial x_{20}^2}\right) & \left(\frac{\partial^2 F}{\partial x_{20}\partial x_{30}}\right) & \cdots & \left(\frac{\partial^2 F}{\partial x_{20}\partial x_{n0}}\right) \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \left(\frac{\partial^2 F}{\partial x_{n0}\partial x_{10}}\right) & \left(\frac{\partial^2 F}{\partial x_{n0}\partial x_{20}}\right) & \left(\frac{\partial^2 F}{\partial x_{n0}\partial x_{30}}\right) & \cdots & \left(\frac{\partial^2 F}{\partial x_{n0}^2}\right) \end{vmatrix} > 0. \quad (3a)$$

This means that all the determinants marked by the dotted lines must be simultaneously positive. The conditions (b) contains in itself the condition that all the second partial derivatives must be positive:

$$\text{(c)} \quad \left(\frac{\partial^2 F}{\partial x_{10}^2}\right) > 0, \quad \left(\frac{\partial^2 F}{\partial x_{20}^2}\right) > 0, \quad \cdots \left(\frac{\partial^2 F}{\partial x_{n0}^2}\right) > 0. \quad (3b)$$

The above given conditions should be fulfilled for a relative as well as for an absolute minimum.

4. According to the generalized Hooke's Law, each of the six independent components of stress at any point of an elastic solid body are connected with the six components of strain at the point by equations of the type [6] (called sometimes stress-strain equations):

$$\begin{aligned} X_x &= c_{11}e_{xx} + c_{12}e_{yy} + c_{13}e_{zz} + c_{14}e_{yz} + c_{15}e_{zx} + c_{16}e_{xy}, \\ &\cdots\cdots \\ Y_z &= c_{41}e_{xx} + c_{42}e_{yy} + c_{43}e_{zz} + c_{44}e_{yz} + c_{45}e_{zx} + c_{46}e_{xy}, \\ &\cdots\cdots \end{aligned} \quad (4)$$

Conversely, we may formulate the six strain-stress equations of the type:

$$e_{xx} = d_{11}X_x + d_{12}Y_y + d_{13}Z_z + d_{14}Y_z + d_{15}Z_x + d_{16}X_y. \quad (4a)$$

When the material is isotropic, we have the following stress-strain and strain-stress relations:

$$\begin{aligned} &X_x = \lambda\Delta + 2\mu e_{xx}, \quad Y_y = \lambda\Delta + 2\mu e_{yy}, \quad Z_z = \lambda\Delta + 2\mu e_{zz}, \\ &Y_z = \mu e_{yz}, \qquad Z_x = \mu e_{zx}, \qquad X_y = \mu e_{xy}. \end{aligned} \quad (5)$$

Solving these we have:

$$
\begin{aligned}
e_{xx} &= \frac{1}{E}\{X_x - \sigma(Y_y + Z_z)\},\\
e_{yy} &= \frac{1}{E}\{Y_y - \sigma(Z_z + X_x)\},\\
e_{zz} &= \frac{1}{E}\{Z_z - \sigma(X_x + Y_y)\},\\
e_{yz} &= \frac{2(1+\sigma)}{E} Y_z,\\
e_{zx} &= \frac{2(1+\sigma)}{E} Z_x,\\
e_{xy} &= \frac{2(1+\sigma)}{E} X_y,
\end{aligned}
\tag{6}
$$

where

$$
E = \frac{\mu(3\lambda + 2\mu)}{(\lambda + \mu)}, \qquad \sigma = \frac{\lambda}{2(\lambda + \mu)}, \qquad \Delta = e_{xx} + e_{yy} + e_{zz}.
$$

The quantity E is "Young's modulus," the number σ is "Poisson's ratio," the quantity μ is the rigidity, the quantity $(\lambda + \frac{2}{3}\mu) = k$ is the "modulus of compression."

5. Let the function W, representing potential energy per unit volume stored up in the body by the strain, be called a strain-energy function. The total potential energy, stored in a body, entirely depends upon the strained shape of the body and is called the elastic strain-energy. Under conventional assumptions the strain-energy function is a homogeneous quadratic function of the strain components [6]. The expression for $2W$ may be written in the form:

$$
\begin{aligned}
2W = {} & c_{11}e_{xx}^2 + 2c_{12}e_{xx}e_{yy} + 2c_{13}e_{xx}e_{zz} + 2c_{14}e_{xx}e_{yz} + 2c_{15}e_{xx}e_{zx} + 2c_{16}e_{xx}e_{xy}\\
& + c_{22}e_{yy}^2 + 2c_{23}e_{yy}e_{zz} + 2c_{24}e_{yy}e_{yz} + 2c_{25}e_{yy}e_{zx} + 2c_{26}e_{yy}e_{xy}\\
& + c_{33}e_{zz}^2 + 2c_{34}e_{zz}e_{yz} + 2c_{35}e_{zz}e_{zx} + 2c_{36}e_{zz}e_{xy}\\
& + c_{44}e_{yz}^2 + 2c_{45}e_{yz}e_{zx} + 2c_{46}e_{yz}e_{xy}\\
& + c_{55}e_{zx}^2 + 2c_{56}e_{zx}e_{xy}\\
& + c_{66}e_{xy}^2.
\end{aligned}
\tag{7}
$$

We may easily express $2W$ as a function of stress components:

$$
2W = C_{11}X_x^2 + 2C_{12}X_xY_y + 2C_{13}X_xZ_z + 2C_{14}X_xY_z + \cdots + \cdots + \cdots. \tag{7a}
$$

6. The equations of equilibrium of a body which is at rest under

the action of body forces and surface tractions may be given in the form:

$$\begin{aligned}
\frac{\partial}{\partial x}\left(\frac{\partial W}{\partial e_{xx}}\right)+\frac{\partial}{\partial y}\left(\frac{\partial W}{\partial e_{xy}}\right)+\frac{\partial}{\partial z}\left(\frac{\partial W}{\partial e_{xz}}\right)+\rho\bar{X}&=0,\\
\frac{\partial}{\partial x}\left(\frac{\partial W}{\partial e_{xy}}\right)+\frac{\partial}{\partial y}\left(\frac{\partial W}{\partial e_{yy}}\right)+\frac{\partial}{\partial z}\left(\frac{\partial W}{\partial e_{yz}}\right)+\rho\bar{Y}&=0,\\
\frac{\partial}{\partial x}\left(\frac{\partial W}{\partial e_{xz}}\right)+\frac{\partial}{\partial y}\left(\frac{\partial W}{\partial e_{yz}}\right)+\frac{\partial}{\partial z}\left(\frac{\partial W}{\partial e_{zz}}\right)+\rho\bar{Z}&=0,
\end{aligned} \tag{8}$$

where $\bar{X}$, $\bar{Y}$, $\bar{Z}$ are the components of the body force per unit of mass and ρ is the density.

7. The total elastic strain energy of a body, which is in equilibrium under a given load, is equal to the work done by the external forces. Hence it may be calculated from the formula:

$$V=\tfrac{1}{2}[X_1\delta_1+X_2\delta_2+X_3\delta_3+\cdots], \tag{9}$$

where X_r is the external force and δ_r the corresponding displacement. Under the conventional assumptions the value of V may be expressed as a homogeneous quadratic function of the external forces:

$$V=\tfrac{1}{2}\sum_{ii=11}^{nn}a_{ii}X_i^2+\sum_{ij=12}^{nm}a_{ij}X_iX_j, \tag{10}$$

where the second sum contains all possible combinations X_iX_j except those with equal subscripts. The other form of expressing V is the following:

$$V=\int\int\int W dxdydz. \tag{11}$$

We may express W by the stress components, and the stress components by Airy's stress function:

$$X_x=\frac{\partial^2\Phi}{\partial y^2},\qquad Y_y=\frac{\partial^2\Phi}{\partial x^2},\qquad \cdots X_y=\frac{\partial^2\Phi}{\partial x\partial y}. \tag{12}$$

Hence we may use the formula:

$$V=\int\int\int F\left[\left(\frac{\partial^2\Phi}{\partial x^2}\right),\left(\frac{\partial^2\Phi}{\partial y^2}\right),\cdots\left(\frac{\partial^2\Phi}{\partial x\partial y}\right),\cdots\right]dxdydz. \tag{13}$$

Still another way of expressing the stresses is the use of Maxwell's stress functions or Morera's stress functions. The general form of the presentation of the total strain-energy in these cases will not be changed.

8. Below we shall cite three important theorems expressing the notion of the elastic strain energy [6], [12].

(a) First Theorem of Castigliano.

The displacement corresponding with any one (T_r, say) of a system of forces $T_1, T_2, \cdots T_n$ can be found by forming an expression for the total elastic strain-energy in terms of the forces, and taking its partial differential coefficient with respect to T_r.

$$\frac{\partial V}{\partial T_1} = \lambda_1, \qquad \frac{\partial V}{\partial T_2} = \lambda_2, \qquad \cdots \frac{\partial V}{\partial T_n} = \lambda_n. \tag{14}$$

In particular, if T_r is the force exerted by a constraint at the point numbered r, the partial differential coefficient with respect to T_r will vanish.

The theorem complementary to the given theorem may be formed by interchanging the words displacement and force.

(b) First Theorem of Minimum Strain-Energy.

Consider the case when there are no body forces and the surface displacements are given. The theorem may be stated in the form [6], [12]:

When specified displacements are imposed upon a body which obeys Hooke's law, satisfying all the conditions at the bounding surface, the distortion which occurs in the configuration of equilibrium involves less strain-energy than any other configuration (not satisfying the conditions of equilibrium) in which the imposed displacements have the same specified values and satisfy the same conditions at the bounding surface.

The converse of this theorem has been employed by Lord Kelvin to prove that there exists a unique solution of the equations of equilibrium which yields given values for the displacements at the boundary.* However, one may find in the literature some doubts expressed concerning the truth of this converse theorem.†

(c) Second Theorem of Minimum Strain-Energy (Castigliano's Second Theorem or Theorem of Least Work).

We assume that specified forces are imposed at certain points in the body and we consider only such configurations as can be maintained in equilibrium by those forces. The theorem states [12]: The value of the total strain-energy when specified external forces are operative is in-

* Reference [7], p. 614, §67: "It is now easy to prove directly, for the case of infinitely small strains, that the solution of the equations of interior equilibrium, whether for a heterogeneous or a homogeneous solid, subject to the prescribed surface conditions, is unique." Below he says, "But (as has been remarked by Professor Stokes to the author) when the surface data are of force, not of displacement, or when force acts from without, on the interior substance of the body, the solution is not in general unique, and there may be configurations of unstable equilibrium, even with infinitely small displacement."

† Love in Ref. [6], p. 172, says, "If we knew independently that among all the sets of functions, which take the given values on the boundary, there must be one which gives a smaller value to V than any other gives, we could infer the truth of this converse theorem."

creased by self-straining of any kind whatever, and assumes its minimum value when there is no self-strain.‡

Hence the differences between various configurations can come only from differences in the initial self-straining. The total elastic strain-energy will always be increased by self-straining, and accordingly is least in that configuration which involves no initial self-strain [12].

Let us cite below some remarks referring to this last theorem according to Ref. [12].§ Let λ_1 stand for the distance apart of two points in a body which has been relieved of self-straining by means of suitable cuts; and let T_1 stand for the tension which must be applied at these points in order to close the gap and so restore the condition of self-straining. Let $T_2, T_3, \cdots T_n$ stand for other tensions of the same kind, and $\lambda_1, \lambda_2, \cdots \lambda_n$ for the distances apart of the pairs of points which they have to bring into contact; and let $X_1, X_2, \cdots$ etc., denote the specified system of external forces. We may regard the initially self-strained body subjected to forces $X_1, X_2, \cdots$, etc., as equivalent to an initially unstrained body subjected to $X_1, X_2, \cdots$, etc., acting simultaneously with the tensions $T_1, T_2, \cdots T_n$, if we impose the condition that the displacements corresponding to the tensions have the magnitudes $\lambda_1, \lambda_2, \cdots \lambda_n$. Seen from this standpoint, the tensions are external forces; and if V is the total elastic strain-energy stored in the body, we have by Castigliano's First Theorem the formula (14). Accordingly the distortion will adjust itself so that the strain-energy (which has a definite value corresponding to any specified values of $T_1, T_2, \cdots T_n$) satisfies the differential relations (14). When $\lambda_1, \lambda_2, \cdots \lambda_n$ are all zero, so that the body under consideration is not self-strained, these relations are replaced by

$$\frac{\partial V}{\partial T_1} = \frac{\partial V}{\partial T_2} = \cdots = \frac{\partial V}{\partial T_n} = 0 \qquad (15)$$

and can be expressed in the statement that the strain-energy has a stationary value for small variations in $T_1, T_2, \cdots T_n$. That the stationary value is a minimum value is evident if we reflect that no upper limit is imposed upon the magnitude of V.

Hence according to this principle the stress distribution resulting from given forces, applied to a body initially in a state of ease, can be deduced from the conditions of equilibrium combined with the conditions for a *minimum* value of the total strain-energy V.*

‡ A body may be "self-strained," in the sense that energy may be stored in it although no forces are operative, and the energy could be recovered if the body were suitably cut.

§ P. 96, §91.

* Ref. 13, p. 212, 3. See also other paper of R. V. Southwell, *Proc. Roy. Soc.* (A), Vol. 154 (1936), pp. 4–21. The last theorem sometimes has the form: Among all systems of stress resultants which are in equilibrium with the given loads and such reactions as the supports can exert on the structure, the actual one minimizes the complementary energy. See: A.

Let us point out that in all three theorems cited above the value of the total strain-energy V corresponds with the stationary value of the strain-energy in the configuration of equilibrium, i.e., with a minimum value of the strain-energy (absolute minimum). No special attention should be confined to concentrated forces; distributed loadings can also be brought within the scope of those theorems [12].

9. Below we shall present the so-called "method of least work" as it finds application in the solution of practical problems following exactly the method of a book on the subject of theory of structures.

Concerning the applications of Castigliano's theorem to statically determinate systems we found that the displacement of any point is obtained as the derivative of the strain energy of the system with respect to the corresponding force. Applying the theorem in the same way to a statically indeterminate system, we conclude that the derivative of the strain energy with respect to any redundant reaction or internal constraint must be zero since it is the function of such a reactive force to prevent any displacement at its point of application.* Hence the

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Castigliano, "Nuovo teoria intorno all'equilibrio dei sistemi elastici," Torino, Atti della Accademia delle Scienze, pp. 127–286, 1876. Also: F. Engesser: Über statisch unbestimmte Tragwerke bei beliebigem Formänderungsgesetze und über den Satz von der kleinsten Ergänzungsarbeit," *Z. Arch. u. Ing.-Vereins zu Hannover*, 35, p. 733, 1889.

* In a book on the subject of elasticity we read: "In considering structures with superfluous reactive elements it will always be assumed that the superfluous fastenings are removed and that corresponding superfluous reactive elements X, Y, Z are applied to the structure. If the superfluous fastenings are such that there are no displacements corresponding to the forces X, Y, Z then from Castigliano's theorem it follows that $\frac{\partial V}{\partial X} = \frac{\partial V}{\partial Y} = \frac{\partial V}{\partial Z} = 0$ (a). The number of these equations is equal to the number of superfluous reactive elements; all these unknown quantities can be found from equations (a). It is easy to show that the second derivative of V with respect to any external force always will be positive. Due to the fact that the function $V = aX^2 + bY^2 + cZ^2 + dXY + \cdots$ must be positive for all values of X, Y, Z, $\cdots$, the coefficients a, b, c, $\cdots$ for X^2, Y^2, Z^2, $\cdots$ must be positive; therefore the second derivatives of V with respect to X, Y, Z, $\cdots$ must be positive. Hence the previous equations (a) represent conditions of minimum V. It follows, therefore, that the magnitudes of the superfluous reactive elements are such as to reduce the potential energy of the system to a minimum value."

In another book on the theory of elasticity we read: "If we have a body with given forces acting on the boundary, and if we consider such changes of stress components as do not affect the equations of equilibrium and the boundary conditions, the true stress components are those making the variation of strain energy vanish. It can be shown that these correct values of the stress components make the strain energy a minimum. Then $\partial V = 0$ expresses the so-called principle of least work. This equation holds also if a portion of the boundary is held rigidly fixed by the constraints and the changes of stress components are such that there are variations of surface forces along this constrained portion of the boundary. Since the displacement along the constrained boundary is zero, we arrive again at $\partial V = 0$. If X, Y, Z, $\cdots$ are forces or couples acting in redundant elements or at redundant constraints of an elastic system, the magnitudes of these statically indeterminate quantities can be calculated from the condition that the strain energy of the system, represented as a function of X, Y, Z, $\cdots$ must be a minimum, i.e. we have the equations: $\frac{\partial V}{\partial X} = 0$, $\frac{\partial V}{\partial Y} = 0$, $\cdots$.

magnitudes of redundant reactions in statically indeterminate systems will be such as to make the system a maximum or minimum. By calculating the second derivatives and proving that they are always positive, it can be shown that we have the case of a minimum. Thus we obtain a method of calculation of redundant forces that is known as the method of least work. We derive the expression for the strain energy of the given system as a function of the redundant forces and then select these forces so as to make this strain energy a minimum. The method of least work can be applied also to structures having several redundant reactions $X, Y, Z, \cdots$. Observing that the supports either are immovable or move perpendicularly to the reactions, we conclude, from Castigliano's theorem, that $\frac{\partial V}{\partial X} = 0, \frac{\partial V}{\partial Y} = 0, \frac{\partial V}{\partial Z} = 0, \cdots$. Thus we obtain as many additional equations as there are redundant reactions and these equations state that the redundant reactions $X, Y, Z, \cdots$ must have such magnitudes as to make the strain energy stored in the structure a minimum. The foregoing conclusion holds also in cases where the redundant reactions are presented by generalized forces.

10. In this paragraph we shall present another application of the principle of least work to the solution of practical problems. Let V be expressed by formulas (11), (12), or (13) and let us assume that the stress-components or Airy's stress function are unknown. The usual procedure in this case is the following:

(i) Express the stress-components or Airy's stress function as certain functions of the coordinates in the chosen system of coordinates $[x(t), y(t), z(t), t]$, for example, and of constant coefficients undetermined for the time being:

$$\begin{aligned} X_x &= X_x(a_0, a_1, a_2, \cdots a_n, \cdots x, y, z, t), \\ &\cdots\cdots \\ Y_x &= \cdots \\ &\cdots\cdots \\ \Phi &= \Phi(a_0, a_1, a_2, \cdots a_n, \cdots x, y, z, t). \end{aligned} \tag{16}$$

The chosen functions must fulfill all the boundary conditions. Various forms of these functions may be chosen:

$$\begin{aligned} X_x &= \psi_0 + a_1\psi_1 + a_2\psi_2 + \cdots, & \psi_n &= \psi_n(x, y, z, t), \\ \Phi &= \Phi_0 + a_1\Phi_1 + a_2\Phi_2 + \cdots, & \Phi_n &= \Phi_n(x, y, z, t). \end{aligned} \tag{17}$$

Power series and Fourier series are functions very often used.

(ii) In the next step the total strain-energy of the body is calculated and formulated as a function of the indetermined coefficients $a_1, a_2, \cdots a_n, \cdots$.

(iii) The equilibrium conditions are taken into account so as to

present the constraining conditions for a minimum of a function (call them Λ).

(iv) The unknown coefficients a_n are determined from the set of linear equations:

$$(a)\quad \frac{\partial}{\partial a_n}\int\int\int W dxdydz + \lambda\frac{\partial \Lambda}{\partial a_n} = 0, \qquad \Lambda = 0,$$

or

$$(a)\quad \frac{\partial}{\partial a_n}\int\int\int F\left[\frac{\partial^2\Phi}{\partial x^2}, \frac{\partial^2\Phi}{\partial y^2}, \cdots\right] dxdydz + \lambda\frac{\partial \Lambda}{\partial a_n} = 0, \Lambda = 0. \qquad (18)$$

11. We shall show that the presentation of the so-called "least work method" and the interpretation of the possibility of the application of Castigliano's theorem as given in § 9 and § 10, may lead to approximate results which are too far from the results in reality to be acceptable. Besides that the presentation cited above does not involve the absolute minimum of the energy and consequently does not represent reasonably Castigliano's theorem. Let us consider a statically indeterminate elastic system or an elastic body which is held stationary by a self-equilibrating system of generalized forces $X_1, X_2, X_3, \cdots$, etc., in which we include the reactions exerted at points of supports. The system or the body are initially in a state of ease. The statically indeterminate values in the system or the stress distribution in the body resulting from the given generalized forces are unknown. They ought to be calculated by the use of the principle of least work. Let us represent the statically indetermined generalized forces in the system or the unknown stresses in the body as certain functions, as explained in § 9 and § 10, and let us express the total strain-energy due to the deformation as a function of the redundant generalized forces or as a function of the unknown constant coefficients. The equating to zero of the first partial derivatives of the strain energy taken with respect to the unknown values does not always solve the problem in a satisfactory way. We pointed out above that in the theorems containing the notion of the strain-energy the value of the strain-energy corresponds always with the stationary value which is an absolute minimum. Equating to zero the first partial derivatives of the function representing the total strain-energy of the body taken with respect to the unknown values and fulfilling all the conditions (3) in order that the quadratic form be a definite positive, may give us a minimum of the function which may not coincide with the absolute minimum of the strain-energy in the actual conditions. The discrepancy between the dependent and independent variables for the absolute minimum in the actual conditions on one side and the dependent and independent variables for the calculated minimum on the other side may be too large to be neglected (see Appendix). Conse-

quently, in order to be able to apply successfully and infallibly Castigliano's theorem to specific problems, we have to be assured a priori, that the redundant generalized forces or unknown stress distributions are expressed by such functions or such sets of functions, among which there is one and only one function or set of functions which gives a smaller value to V than any other gives, having first taken the given values on the boundary (i.e., existence and uniqueness of an absolute minimum must be assured). Being assured that such an absolute minimum does exist and is unique, we present the strain-energy as a function of the redundant generalized forces or of the unknown coefficients and next we express in an analytical form the statement that the strain-energy has a stationary value for small variations in $T_1, T_2, \cdots T_n$, by equating to zero the first partial derivatives of V with respect to $T_1, T_2, \cdots T_n$. This gives the correct solution.

In practice this means that application of any arbitrary function may give results lying so far from reality that they cannot even be considered to be approximate. Hence in order to obtain approximately correct results one has to apply in each particular case a particular function.* The problem how to find a proper function in each particular case is not yet solved in a rigorous way. One has to depend on the intelligent guess or on his experience.

The above given presentation of the procedure based upon the principle of least work or the so-called "least work method" is different from those cited in § 9. From this presentation it is obvious that the procedures described in § 9 and § 10 cannot always be successful and what is more, it explains when those procedures may be successful. When we apply these procedures to the calculation of the redundant generalized forces which can be analytically expressed in one and only one way, the procedure is usually successful and we are justified to call it a method.† When we apply this procedure to the calculation of the redundant generalized forces or stresses which are expressed as certain functions with unknown coefficients, without an assurance a priori that among the applied functions or sets of functions there is one and only one which gives a least value to V, then the procedure may not be successful. Hence, it seems rather doubtful whether in these cases the procedure

* As an example we may cite that Fourier series proved to be a very successful function in the theory of plates. But this does not mean that it must be a proper function in any other type of structure.

† This case occurs when we apply the theorem to the calculation of redundant generalized forces in beams, frameworks, rings, etc. In these elastic systems the generalized forces are usually expressed not as functions of coordinates, but as pure numbers which are unknown at the beginning, and the strain-energies due to these generalized forces can be expressed in a unique way. [Strain energy in tension: $V = (\frac{1}{2})W^2L/(EA)$; in a bent beam: $V = \int_0^L (M^2/(2EI))dx$; in a bar in simple torsion: $V = \int_0^L (T^2/(2GC))dx$, etc.].

may be considered even as an approximate method.* This is a random-choice method or procedure. There are cases in the literature ([9], [10], [14]) in which the procedure gave good approximate results but there are also cases in which the procedure fails completely.

12. On the basis of the preceding remarks the following conclusions can be derived:

(i) In order to be able to apply successfully and infallibly Castigliano's theorem in specific practical problems, we have to be assured a priori, that the redundant generalized forces or unknown stress distributions are expressed by such functions or such sets of functions, taking the given values on the boundary, among which there is one and only one function or set of functions which gives a smaller value to V than any other gives. The problem, how to find such a function in any particular case, is not yet solved in a rigorous way.

(ii) When we apply the procedure based upon the "principle of least work" or the "least work method" to the calculation of the redundant generalized forces which can be analytically expressed in one and only one way, the procedure is usually successful.

(iii) When we apply this procedure to the calculation of the redundant generalized forces or stresses which are expressed as certain functions with unknown coefficients, without an assurance a priori that among the applied functions there exists one and only one which gives a least value to V, then the procedure may give the results lying too far from the actual conditions to be reliable. On the basis of these conclusions the author proposes the presentation of the least work method as given at the beginning of this paper.† The existing presentation of the so-called principle of least work method may be considered to be correct but only in certain particular cases.

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* This may happen when the procedure described in §10 is applied.

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APPENDIX.

In order to check the theoretical results a numerical example was calculated; namely, the normal stresses in a thin-walled rectangular two-field box were found (Fig. 1). The box consisted of sheets carrying only normal stresses, of the covering skin carrying only shearing stresses and of a certain number of reinforcing ribs. The vertical webs of the box were supposed to carry shear stresses only and no normal stresses. All these assumptions are in agreement with the general practice met in aircraft stress analysis. The shear flow in the ith field was represented by the formula:

$$q_i = \sum_{n=1}^{\infty} a_{in} \sin\left(\frac{n\xi\pi}{2a}\right) \tag{a}$$

where the used symbols denote:

a_{in} = Fourier coefficient,
n = integer $(1, 2, 3, \cdots n)$,
ξ = Running coordinate along the circumference composed of two longer sides of the rectangle, omitting the vertical webs (from 0° to 180° in Fig. 1),
$2a$ = length of the longer side of the rectangle.

From the conventional relations the normal stresses were derived and the total strain energy of the loaded box was calculated including the strain energy of the ribs. The acting load was assumed to be a concentrated transverse force P attached in the plane of symmetry of the first rib. The unknown Fourier coefficients were calculated with the application of the so-called principle of least work method from the set of

linear equations of the form:

$$\frac{\partial V_{tot}}{\partial a_{in}} + \frac{\lambda \partial \Lambda}{\partial a_{in}} = 0, \qquad \Lambda = 0, \tag{b}$$

where the symbol denotes:

V_{tot} = total strain energy of the loaded box,
Λ = constraining function,
λ = Lagrangian multiplier.

Following the rule based upon the mathematical induction that if a statement is valid up to the limit it is also valid in the limit, the next assumption made was, namely, that the stiffness of the ribs is infinitely great. The stress distribution in this type of structure is very well known. Fig. 2 shows the distribution of the normal stresses presented in a dimensionless form $\left[\frac{\sigma_p a^2 t_e}{PL}\right]$ (σ_p = normal stress) as a function of the running coordinate. The dotted line in Fig. 2 shows the dimensionless stress distribution calculated from the conventional formula:

$$\sigma_p = \frac{My}{I}, \tag{c}$$

where the used symbols denote:

M = bending moment,
y = distance from the neutral axis,
I = moment of inertia,
t_e = thickness of sheet carrying normal stresses,
P = acting load,
L = arm of the bending moment.

We see that the stresses calculated with the application of the so-called principle of least work method are in many points greater than the stresses obtained by use of the conventional formula. Moreover, in certain points of the compression side there are tensile stresses, and on the tensile side, compressive stresses. Besides that the value of stresses for 90° drops down to zero which is in contradiction to all the results of practical experiments. The difference in areas under the curved line, taken with the corresponding signs, is equal on each side of the box to the area under the dotted line (concerning the absolute value and the sign). All this means that the value of the total strain energy used in the so-called principle of least work method at the point obtained by equating to zero the first partial derivatives taken with respect to the unknown Fourier coefficients does not represent the absolute minimum of the total strain energy which occurs in actual conditions. The value of the strain energy used in calculation is greater than the value of the actual strain-energy.

Taking into account five Fourier coefficients in each field, the matrix in the calculated example, arranged according to the scheme:

$$\begin{vmatrix} \frac{\partial^2 V_{tot}}{\partial a_{11}^2} & \frac{\partial^2 V_{tot}}{\partial a_{11}\partial a_{12}} & \frac{\partial^2 V_{tot}}{\partial a_{11}\partial a_{13}} \cdots & \frac{\partial^2 V_{tot}}{\partial a_{11}\partial a_{21}} & \frac{\partial^2 V_{tot}}{\partial a_{11}\partial a_{22}} \cdots \\ \frac{\partial^2 V_{tot}}{\partial a_{12}\partial a_{11}} & \frac{\partial^2 V_{tot}}{\partial a_{12}^2} & \frac{\partial^2 V_{tot}}{\partial a_{12}\partial a_{13}} \cdots & \frac{\partial^2 V_{tot}}{\partial a_{12}\partial a_{21}} & \cdots\cdots\cdots \\ \vdots & \vdots & \vdots & \vdots & \\ \frac{\partial^2 V_{tot}}{\partial a_{15}\partial a_{11}} & \cdots\cdots & & & \\ \frac{\partial^2 V_{tot}}{\partial a_{21}\partial a_{11}} & \cdots\cdots & & & \\ \vdots & & & & \\ \frac{\partial^2 V_{tot}}{\partial a_{25}\partial a_{11}} & & & & \end{vmatrix} \tag{e}$$

looks:

$$\begin{vmatrix} 0.0115 & 0 & 0 & 0 & 0 & 0.0005 & 0 & 0 & 0 & 0 \\ 0 & 0.0160 & 0 & 0 & 0 & 0 & 0.0020 & 0 & 0 & 0 \\ 0 & 0 & 0.0235 & 0 & 0 & 0 & 0 & 0.0045 & 0 & 0 \\ 0 & 0 & 0 & 0.0340 & 0 & 0 & 0 & 0 & 0.0080 & 0 \\ 0 & 0 & 0 & 0 & 0.0475 & 0 & 0 & 0 & 0 & 0.0125 \\ 0.0115 & 0 & 0 & 0 & 0 & 0.0103 & 0 & 0 & 0 & 0 \\ 0 & 0.0060 & 0 & 0 & 0 & 0 & 0.0113 & 0 & 0 & 0 \\ 0 & 0 & 0.0135 & 0 & 0 & 0 & 0 & 0.0130 & 0 & 0 \\ 0 & 0 & 0 & 0.0240 & 0 & 0 & 0 & 0 & 0.0153 & 0 \\ 0 & 0 & 0 & 0 & 0.0375 & 0 & 0 & 0 & 0 & 0.0183 \end{vmatrix} \tag{f}$$

The obtained value of the function is a minimum since the matrix is definite positive. Hence, the calculation proved the theoretical considerations. In another example a double Fourier Series was applied to the calculation of stresses caused by pure bending moment in the vertical plane in a circular monocoque cylinder with a cut-out. The normal stresses in the full shell part were expressed by the formula:

$$\sigma_2 = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} b_{mn} \cos \frac{m(x - L_1)\pi}{L_2} \cos n\phi, \tag{g}$$

where the symbols denote:

x—coordinate along the axis of the circular monocoque,
L_1—half of the length of the cut-out,
L_2—length of the full shell part,
ϕ—angle measured along the perimeter of the circle.

The normal stresses in the cut-out part were expressed by the formula:

$$\sigma_1 = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} a_{mn} \cos \frac{mx\pi}{L_1} \cos n\phi. \tag{h}$$

Next the shearing stresses were calculated and the total strain-energy of the system was expressed as the function of the unknown Fourier coefficients. Seventeen coefficients were taken into account. Together with Lagrangian multipliers (constraining condition) they led to a system of 26 linear equations. The values of the coefficients in p.s.i. are as follows:

$$\begin{array}{lll} a_{00} = 313472.10, & a_{01} = 516715.23, & a_{02} = 221147.89, \\ a_{10} = 0, & a_{11} = 0, & a_{12} = 28330.76, \\ a_{20} = -313472.10, & a_{21} = -492154.33, & a_{22} = -218819.90, \end{array} \tag{i}$$

and

$$\begin{array}{llll} b_{00} = 0, & b_{01} = 24560.90, & b_{02} = -10617.75, & b_{03} = -10754.86, \\ b_{10} = 0, & b_{11} = 0, & b_{12} = -9800.19, & b_{13} = -14189.96, \\ b_{20} = 0, & b_{21} = 0, & b_{22} = -3737.45, & b_{23} = -7001.41, \\ b_{30} = 0, & b_{31} = 0, & b_{32} = -1845.36, & b_{33} = -3785.71. \end{array} \tag{j}$$

From the calculated coefficients it is not possible to judge whether the Fourier Series is convergent or divergent in any or both directions. As is known in many physical applications the first few Fourier coefficients increase in their values while the subsequent ones decrease very fast. For this reason more terms should be taken into account. This, on the other hand, would lead to systems of linear equations which are difficult to solve without using special computing machines. However the first few calculated determinants of the matrix, arranged according to the scheme (e), were positive. This fact and the general form of the matrix would possibly show that the matrix is a definite positive one. There is a lack of tests of the construction of the type considered and no comparison can be made. The simple calculations show that the stresses in the end points of the vertical diameter at the end of the full shell part do not differ much from the stresses calculated from the conventional formula (c). The actual stress distribution will certainly differ very much from that one calculated by the formula (c). But the large values of the coefficients for higher values of m and n mean that there may be a great variation in signs of stresses in the stress pattern along the perimeter of a circle in each cross section. This variation does not look quite justified because of simple pure bending moments at both ends and a rigid construction (the assumption was made that the number

of the transverse rings is infinite and that they are of an infinitely great rigidity). On the other hand the strain-energy will be very great because of this fact which again will show that the minimum of the function does not coincide with the minimum of the actual strain energy. This again would prove the theoretical considerations.

THE DETERMINATION OF SULFATE AS BARIUM SULFATE.
(In the Presence of Metals of the Acid Hydrogen Sulfide Group.)

BY

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INTRODUCTION.

The object of this investigation was to determine the accuracy obtained in the precipitation of the sulfate ion as barium sulfate after removal of those metals which are quantitatively precipitated from a dilute hydrochloric acid solution with hydrogen sulfide. A comparison was made of the results obtained after the removal of the metals of the acid hydrogen sulfide group with those which were obtained by direct precipitation of barium sulfate in the presence of the metallic ions. The metals considered were cadmium, mercury, bismuth, copper, antimony, arsenic, and tin.

Lehrman, Been and Mandell (1), in studying the removal of the barium ion with the sulfides of the ammonium sulfide group, have found that oxidation of sulfide to sulfate ion does not occur when these sulfides are precipitated. The loss of barium ion is attributed to surface adsorption and post precipitation. However, Curtman and Frankel (2) have indicated that in the qualitative procedure for metals of Group II there is the possibility of a loss of barium ion due to the formation of sulfate ion by oxidation of hydrogen sulfide.

There are several possible sources of error which may exist in the procedure whereby metals of the acid hydrogen sulfide group are removed prior to the precipitation of the sulfate ion with barium chloride. Oxidation of sulfide to sulfate may be catalyzed in the presence of precipitated sulfides, due to adsorption of oxygen and hydrogen sulfide on the surface of the precipitate. Hydrogen sulfide may be oxidized to sulfate through the action of dissolved oxygen in the solution or by air oxidation during the filtration. The same is true of the sulfides themselves. It is well known that moist copper sulfide is subject to air oxidation under the usual laboratory conditions of filtration. Because of this oxidation the precipitation of copper as the sulfide is not a preferred analytical procedure (3, 4).

Coprecipitation of the sulfate ion with metallic sulfides due to adsorption or compound formation may lead to low results for sulfate. When mercury is precipitated as the sulfide from mercuric chloride solution, high results for mercury have been obtained due to the coprecipitation of chloride (5). It is possible that sulfate ions may be removed

from solution by a similar coprecipitation. Treadwell and Hall (6) state that cadmium sulfide is always contaminated by salts of the type Cd_2Cl_2S when precipitated from acid solution. According to Weiser and Durham (7) the sulfide that is obtained in acid solution is not a double salt but cadmium sulfide contaminated by adsorbed cadmium salt.

The general experimental procedure was to prepare the solutions in such a manner that there was the same amount of added sulfate ion present in each determination. A stock solution of sulfuric acid, containing approximately 0.1 gram of sulfuric acid in 25 ml. of solution was standardized gravimetrically. Portions of this solution were drawn off at 25° C. by means of a pipette calibrated at the same temperature. Solutions of the chlorides of the metals were added in varying concentrations and combinations and removed by precipitation with hydrogen sulfide according to recommended procedures. The precipitates were filtered on paper and washed with water or a saturated solution of hydrogen sulfide when necessary. The filtrates were boiled to expel the hydrogen sulfide and evaporated on a steam bath. The sulfuric acid remaining in the beakers was diluted to a volume of 100 ml. and the sulfate determined as barium sulfate.

One of the chief concerns was to obtain identical conditions and reproducibility of the sulfate determinations after the removal of the metals of the acid hydrogen sulfide group. In accomplishing this, care was taken to adhere closely to the following conditions for the final sulfate analyses:

The amount of foreign ions was reduced to a minimum. No alkali metals were present. Hydrochloric acid was removed by evaporation, and the only possible contaminants were those due to the slight excess of barium chloride and the hydrochloric acid formed by metathesis.

The barium sulfate precipitations were carried out under the same conditions. The volume of solution, concentration of sulfuric acid, temperature of precipitation, duration of time of addition of precipitating agent, concentration of precipitating agent, and the amount of excess precipitant were maintained constant.

Identical conditions of filtration and ignition were observed. The time of digestion, amount of wash liquid used, and the temperature and time of ignition were not varied.

A comparatively small amount (0.24 g.) of barium sulfate was precipitated in an attempt to give definite indications of small deviations in the analyses.

REAGENTS AND MEASURING INSTRUMENTS.

Baker's (c.p.) reagents were used throughout the investigation.

Standard Sulfuric Acid Solution. Approximately 40 ml. of 95% sulfuric acid solution were diluted to 18 liters.

Barium Chloride was twice recrystallized from water. A 10% solution of the salt was prepared.

Solutions of metal ions were prepared so as to contain 20 g. of the metal per liter of solution.

Cadmium. A stock solution of cadmium was prepared by dissolving 19.6 g. of $CdCl_2 \cdot 2H_2O$ in 500 ml. of solution. A 50 ml. portion of the solution gave no test for sulfate with barium chloride.

Mercury. A stock solution of mercury was prepared by dissolving 13.6 grams of recrystallized $HgCl_2$ in 500 ml. of solution. A 50 ml. portion of the solution gave no test for sulfate with barium chloride.

Bismuth. A hydrochloric acid solution containing 15.1 g. of $BiCl_3$ in 500 ml. of solution was prepared. BiOCl was twice precipitated by dilution, filtered, and redissolved. The solution was diluted to 500 ml. and the acid concentration was adjusted to 1 N. A 50-ml. portion of the solution gave no test for sulfate with barium chloride. To insure the sensitivity of this test, two drops of the standard sulfuric acid, containing approximately 0.4 mg. of H_2SO_4, were added to the 50 ml. of test solution. A definite precipitate of barium sulfate was formed. When the solution was filtered, ignited and weighed, the resulting weight of barium sulfate was 0.3 mg.

Copper. A solution of copper was prepared by dissolving 12.6 g. of CuO in 66 ml. of concentrated hydrochloric acid. A small amount of insoluble material was removed by filtration and the solution was diluted to 500 ml. At this volume the solution was 1 N with respect to hydrochloric acid. A 50-ml. portion of the solution gave no test for sulfate with barium chloride. A positive test for small amounts of sulfate was obtained at this acid concentration as shown for bismuth.

Antimony. A solution of antimony was prepared by dissolving 12 g. of Sb_2O_3 in 140 ml. of concentrated hydrochloric acid. Traces of insoluble material were removed by filtration, and the solution was diluted to 500 ml. At this volume the solution was 3 N with respect to hydrochloric acid. A 50-ml. portion of solution gave no test for sulfate with barium chloride. A precipitate of barium sulfate was formed when two drops of the dilute sulfuric acid, containing approximately 0.4 mg. of H_2SO_4, were added to the 50-ml. portion of test solution containing barium chloride.

Arsenic. A solution of arsenic was prepared by dissolving 13.2 g. of As_2O_3 in 113 ml. of concentrated hydrochloric acid. After being diluted to 500 ml. the solution was 2 N with respect to hydrochloric acid. A 50-ml. portion of the solution gave no test for barium chloride. A positive test for small amounts of sulfate was obtained at this acid concentration as shown for bismuth and antimony.

Tin. A solution of tin was prepared by dissolving 19 g. of $SnCl_2 \cdot 2H_2O$, which had been twice recrystallized from concentrated hydro-

chloric acid, in 40 ml. of concentrated hydrochloric acid and diluting to 500 ml. At this volume the solution was 1 N with respect to hydrochloric acid. A 50-ml. portion of the solution gave no test for sulfate with barium chloride. A positive test for small amounts of sulfate was obtained at this acid concentration as shown for bismuth.

Hydrochloric Acid. All hydrochloric acid used was tested for the absence of sulfate ion with barium chloride.

Hydrogen Sulfide. The hydrogen sulfide used was generated by the action of hydrochloric acid on pyrites in a Kipp generator, and the gas was washed by passing it through distilled water.

Distilled Water. All distilled water used was tested for the absence of sulfate ion with barium chloride.

Filter Paper. Munktell's No. 00 quantitative filter paper was used throughout. The ash content of this paper is 0.03 mg.

Weights. All weighings were made with a set of weights calibrated by the method of T. W. Richards (8).

Volumetric Apparatus. All measuring instruments were calibrated to deliver at 25° C.

PROCEDURE FOR THE DETERMINATION OF SULFATE.

The procedure followed for the determination of the sulfate ion as barium sulfate before and after the removal of the metals of the acid hydrogen sulfide group is essentially that of Richards and Parker (9) as recommended by Hillebrand and Lundell (10). One hundred milliliters of a solution containing approximately 0.1 g. of sulfuric acid are heated

TABLE I.

Standardization of Sulfate Solution.

Date	$BaSO_4$ Found Gram	H_2SO_4 Found Gram
4-19-40	0.2401	0.1009
	0.2400	0.1008
	0.2403	0.1009
	0.2401	0.1009
4-21-40	0.2402	0.1009
	0.2400	0.1008
	0.2400	0.1008
	0.2403	0.1009
11-9-40	0.2399	0.1008
	0.2400	0.1008
	0.2403	0.1009
3-10-41	0.2400	0.1008
	0.2398	0.1007
	0.2400	0.1008
	0.2401	0.1009
Average	0.2401	0.1009

TABLE II.

Determination of Sulfate in the Presence of Hydrochloric Acid.
Calculated Weight of $BaSO_4$ = 0.2401 g.

Series	HCl Initial Conc.	HCl Conc. $BaSO_4$ Precipitation	Time of Digestion	$BaSO_4$ Found	Deviation
	N	N	Hours	Gram	%
1	1.0	0.02	18	0.2402	
				0.2403	
				0.2401	
			Average	0.2402	+<0.05
2	0.15	0.17	18	0.2394	
				0.2390	
				0.2394	
				0.2393	
				0.2396	
			Average	0.2393	−0.3
3	0.32	0.34	80	0.2382	
				0.2382	
			Average	0.2382	−0.8
4	0.90	0.92	168	0.2397	
				0.2396	
			Average	0.2397	−0.2

to boiling and 3 ml. of a 10% barium chloride solution are added dropwise from a burette over a period of one minute while the solution is being stirred. The precipitate is then digested on a steam bath for 18 hours. The solution is filtered through Munktell's No. 00 filter paper and washed with 8–10 small portions of warm water. At this point silver nitrate shows a barely perceptible opalescence when added to 25 ml. of washings. The wet filter paper and precipitate is dried and charred at a low temperature in a weighed porcelain crucible and all carbon is burned off in an oxidizing atmosphere. Finally, the crucible is ignited to constant weight in a furnace at 800° C.

STANDARDIZATION OF SULFATE SOLUTION.

Twenty-five milliliter portions of the standard sulfuric acid solution were transferred to 250-ml. beakers, diluted to 100 ml. and analyzed for sulfate as outlined in the procedure. The results of the analyses as found over a period of time are tabulated in Table I. These analyses show an average deviation of 0.05% from the average value.

EFFECT OF TREATING IGNITED BARIUM SULFATE WITH CONCENTRATED SULFURIC ACID.

Twelve ignited barium sulfate residues chosen at random during the course of the investigation were treated with three drops of concentrated

sulfuric acid. The fumes of SO_3 were carefully expelled and the crucibles again ignited at 800° C. to constant weight. An average increase in weight of 0.03 per cent. was found. This seemed to indicate that no great amount of occluded barium chloride was present in the barium

TABLE III.

Determination of Sulfate After Saturating with Hydrogen Sulfide.
Calculated Weight of $BaSO_4$ = 0.2401 g.

Series	HCl Initial Conc.	Saturated with H_2S for — hr.	HCl Conc. $BaSO_4$ Precipitation	$BaSO_4$ Found	Deviation
	N	Hours	N	Gram	%
5	0.3	0.5	0.02	0.2403	
				0.2401	
				0.2402	
			Average	0.2402	+<0.05
6	0.3	48	0.02	0.2409	
				0.2413	
				0.2415	
				0.2410	
			Average	0.2412	+0.5
7*	0.3	48	0.02	0.2406	
				0.2403	
				0.2410	
				0.2407	
				0.2409	
				0.2403	
			Average	0.2406	+0.2
8	0.3	0.5	0.32	0.2360	
				0.2366	
				0.2365	
			Average	0.2364	−1.5
9**	—	—	—	0.2400	
				0.2403	
				0.2406	
			Average	0.2403	+0.1

* Oxygen removed by passing CO_2 through hot solution.

** Weights of $BaSO_4$ recovered from filtrates of Series 8 after standing for two weeks were added to the weights of $BaSO_4$ from Series 8.

sulfate and that no appreciable reduction of the barium sulfate took place during the charring of the filter paper.

RESULTS.

The sulfate was determined in mixtures of hydrochloric and sulfuric acid. Good values for sulfate were obtained when the hydrochloric acid was expelled by evaporation on a steam bath prior to diluting the

solutions and precipitating the sulfate. Low values for sulfate were obtained when the barium sulfate was precipitated in the presence of hydrochloric acid at concentrations of 0.15 N to 0.9 N. Prolonged digestion of the barium sulfate decreased the error (Table II).

No appreciable oxidation of hydrogen sulfide to sulfate took place when a sulfuric acid solution was saturated with hydrogen sulfide, the

TABLE IV.

Determination of Sulfate Before and After the Removal of Cadmium as the Sulfide.
Calculated Weight of $BaSO_4$ = 0.2401 g.

Series	Cd Present	Cd Removed	HCl Initial Conc.	HCl Conc. $BaSO_4$ Precipitation	$BaSO_4$ Found	Deviation
	Gram		N	N	Gram	%
10	0.1	No	0	0.02	0.2403	
					0.2402	
					0.2400	
				Average	0.2402	+<0.05
11	0.1	Yes	0.3	0.02	0.2403	
					0.2402	
					0.2402	
				Average	0.2402	+<0.05
12*	0.1	Yes	0.3	0.02	0.2408	
					0.2403	
					0.2410	
					0.2404	
				Average	0.2406	+0.2
13	0.02	Yes	0.3	0.02	0.2402	
					0.2401	
				Average	0.2402	+<0.05
14	0.02	Yes	0.3	0.02	0.2409	
					0.2405	
					0.2404	
					0.2410	
					0.2406	
					0.2404	
				Average	0.2406	+0.2

* Precipitated sulfide allowed to stand saturated with hydrogen sulfide for 48 hours before filtering.

solution boiled, evaporated, diluted and the sulfate determined. When the solution was kept saturated with hydrogen sulfide for 48 hours, appreciable oxidation took place. This oxidation was decreased by bubbling an inert gas through the solution prior to saturating with hydrogen sulfide (Table III).

Satisfactory results for barium sulfate were obtained when the

sulfate was precipitated in the presence of cadmium or the cadmium first removed as the sulfide. High results for sulfate were obtained when the cadmium sulfide was allowed to stand saturated with hydrogen sulfide for 48 hours before filtering (Table IV).

Satisfactory results for barium sulfate were obtained when mercury was removed as the sulfide from solutions containing sulfate and the sulfate determined in the filtrate. Sulfate values were slightly high when the barium sulfate was precipitated in the presence of mercury.

TABLE V.

Determination of Sulfate Before and After the Removal of Mercury as the Sulfide.
Calculated Weight of $BaSO_4$ = 0.2401 g.

Series	Hg Present	Hg Removed	HCl Initial Conc.	HCl Conc. $BaSO_4$ Precipitation	$BaSO_4$ Found	Deviation
	Gram		N	N	Gram	%
15	0.1	No	0	0.02	0.2407 0.2406 0.2405	
				Average	0.2406	+0.2
16	0.1	Yes	0.3	0.02	0.2403 0.2401 0.2403	
				Average	0.2402	+<0.05
17*	0.1	Yes	0.3	0.02	0.2414 0.2409 0.2408 0.2412	
				Average	0.2411	+0.4
18	0.02	Yes	0.3	0.02	0.2403 0.2399 0.2400	
				Average	0.2401	+<0.05

* Precipitated sulfide allowed to stand saturated with hydrogen sulfide for 48 hours before filtering.

Allowing the precipitated sulfide to stand for 48 hours gave high results (Table V).

The weights of barium sulfate were 0.4 per cent. high when bismuth sulfide was precipitated by hydrogen sulfide at room temperature from solutions containing sulfate and the sulfate determined in the filtrate. This error was larger when the sulfide was precipitated from hot solution. The error was decreased by bubbling nitrogen through the hot solutions (Table VI).

The sulfate values were slightly low when barium sulfate was pre-

TABLE VI.

Determination of Sulfate Before and After the Removal of Bismuth as the Sulfide.
Calculated Weight of $BaSO_4$ = 0.2401 g.

Series	Bi Present	Bi Removed	Temp. of Sulfide Precipitation	HCl Initial Conc.	HCl Conc. $BaSO_4$ Precipitation	$BaSO_4$ Found	Deviation
	Gram		°C.	N	N	Gram	%
19	0.1	No	25	0.4	0.42	0.2380	
						0.2388	
					Average	0.2384	−0.7
20	0.1	Yes	25	0.3	0.02	0.2411	
						0.2408	
						0.2410	
						0.2410	
					Average	0.2410	+0.4
21	0.1	Yes	90	0.3	0.02	0.2417	
						0.2412	
						0.2418	
						0.2414	
						0.2410	
						0.2416	
					Average	0.2415	+0.6
22*	0.1	Yes	25	0.3	0.02	0.2425	
						0.2422	
						0.2424	
					Average	0.2424	+1.0
23**	0.1	Yes	90	0.3	0.02	0.2409	
						0.2408	
						0.2407	
					Average	0.2408	+0.3
24	0.1	Yes	90	.05	0.07	0.2408	
						0.2410	
						0.2410	
						0.2409	
						0.2412	
					Average	0.2410	+0.4
25	0.02	Yes	90	0.3	0.02	0.2413	
						0.2413	
						0.2411	
						0.2413	
						0.2416	
						0.2411	
					Average	0.2413	+0.5
26	0.01	Yes	90	0.3	0.02	0.2412	
						0.2410	
					Average	0.2411	+0.4

* Precipitated sulfide allowed to stand saturated with hydrogen sulfide for 48 hours before filtering.

** Nitrogen was bubbled through the hot solutions before and after the precipitation of the sulfide.

cipitated in the presence of copper. When copper was removed as the sulfide from solutions containing sulfuric acid, the weights of barium sulfate were high due to the air oxidation of moist copper sulfide (Table VII).

The weights of barium sulfate were 0.4 per cent. high when antimony sulfide was precipitated from hot solutions containing sulfate (Table VIII).

No appreciable error was found when barium sulfate was precipitated from a 0.1 N hydrochloric acid solution containing arsenic as the chlor-

TABLE VII.

Determination of Sulfate Before and After the Removal of Copper as the Sulfide.
Calculated Weight of $BaSO_4$ = 0.2401 g.

Series	Cu Present	Cu Removed	HCl Initial Conc.	HCl Conc. $BaSO_4$ Precipitation	$BaSO_4$ Found	Deviation
	Gram		N	N	Gram	%
27	0.1	No	0.05	0.07	0.2391	
					0.2397	
					0.2397	
					0.2391	
				Average	0.2394	−0.3
28	0.1	Yes	0.3	0.02	0.2479	
					0.2459	
				Average	0.2469	+2.8
29*	0.1	Yes	0.3	0.02	0.2465	
					0.2465	
				Average	0.2465	+2.7
30*	0.02	Yes	0.3	0.02	0.2431	
					0.2423	
					0.2440	
					0.2440	
				Average	0.2434	+1.4

* Nitrogen was bubbled through the hot solutions before and after the precipitation of the sulfide.

ide. When arsenic was first removed as the sulfide, high values for sulfate were obtained and considerable difficulty was encountered in the filtration of the sulfide (Table IX).

The weights of barium sulfate were 0.3 per cent. high when stannous sulfide was precipitated with hydrogen sulfide at room temperature from solutions containing sulfate (Table X).

Correct values for sulfate were obtained when 0.2401 g. of barium sulfate was precipitated directly from a solution containing 0.02 g. each of cadmium, mercury, copper, and arsenic as the chlorides. Weights of

TABLE VIII.

Determination of Sulfate Before and After the Removal of Antimony as the Sulfide.
Calculated Weight of $BaSO_4$ = 0.2401 g.

Series	Sb Present	Sb Removed	HCl Initial Conc.	HCl Conc. $BaSO_4$ Precipitation	$BaSO_4$ Found	Deviation
	Gram		N	N	Gram	%
31	0.1	No	0.9	0.92	0.2380	
					0.2388	
				Average	0.2384	−0.7
32	0.1	Yes	2.5	0.02	0.2410	
					0.2411	
					0.2408	
					0.2410	
					0.2409	
					0.2411	
					0.2409	
				Average	0.2410	+0.4
33	0.02	Yes	2.5	0.02	0.2408	
					0.2406	
					0.2405	
				Average	0.2406	+0.2

barium sulfate were from 0.3 to 0.4 per cent. high when mixed sulfides of the seven metals considered were precipitated and the sulfate in the

TABLE IX.

Determination of Sulfate Before and After the Removal of Arsenic as the Sulfide.
Calculated Weight of $BaSO_4$ = 0.2401 g.

Series	As Present	As Removed	HCl Initial Conc.	HCl Conc. $BaSO_4$ Precipitation	$BaSO_4$ Found	Deviation
	Gram		N	N	Gram	%
34	0.1	No	0.1	0.12	0.2403	
					0.2399	
					0.2399	
				Average	0.2400	−<0.05
35*	0.1	Yes	9.0	0.02	0.2419	
					0.2417	
					0.2417	
					0.2416	
				Average	0.2417	+0.7
36*	0.02	Yes	9.0	0.02	0.2429	
					0.2427	
				Average	0.2428	+1.1

* Nitrogen bubbled through the solutions before and after the precipitation of the sulfide.

TABLE X.

Determination of Sulfate After the Removal of Tin as the Sulfide.
Calculated Weight of $BaSO_4$ = 0.2401 g.

Series	Sn Present	Sn Removed	HCl Initial Conc.	HCl Conc. $BaSO_4$ Precipitation	$BaSO_4$ Found	Deviation
	Gram		N	N	Gram	%
37	0.1	Yes	0.3	0.02	0.2414	
					0.2413	
					0.2414	
					0.2412	
				Average	0.2413	+0.5
38*	0.1	Yes	0.3	0.02	0.2405	
					0.2409	
				Average	0.2407	+0.3
39*	0.02	Yes	0.3	0.02	0.2409	
					0.2411	
				Average	0.2410	+0.4

* Sulfides were filtered in the presence of metallic tin and nitrogen was bubbled through the hot solutions before and after the sulfide precipitations.

TABLE XI.

Determination of Sulfate Before and After the Removal of Metals with Hydrogen Sulfide.
Calculated Weight of $BaSO_4$ = 0.2401 g.

Series	Metals Present	Metals Removed	HCl Initial Conc.	HCl Conc. $BaSO_4$ Precipitation	$BaSO_4$ Found	Deviation
	Gram		N	N	Gram	%
40*	0.14	Yes	0.3	0.02	0.2409	+0.3
41**	0.12	Yes	0.3	0.02	0.2410	
					0.2410	
				Average	0.2410	+0.4
42*	0.7	Yes	0.3	0.02	0.2411	
					0.2410	
					0.2410	
				Average	0.2410	+0.4
43**	0.6	Yes	0.3	0.02	0.2409	+0.3
44***	0.08	No	0.04	0.06	0.2402	
					0.2400	
				Average	0.2401	0.00

*Solutions containing equal weights of Cd, Hg, Bi, Sb, As, Sn, and Cu.
** Solutions containing equal weights of Cd, Hg, Bi, Sb, As, and Sn.
*** Solutions containing equal weights of Cd, Hg, As, and Cu.

filtrate determined. Errors due to oxidation were minimized and difficulties of filtration were eliminated when the mixed sulfides were precipitated (Table XI).

SUMMARY.

Experimental results are given indicating the accuracy obtained when the sulfate ion is determined as barium sulfate before and after the removal of the metals of the acid hydrogen sulfide group from the solution. Solutions of cadmium, mercury, bismuth, copper, antimony, arsenic and tin were used in the determinations.

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NOTE: This paper was abstracted from the dissertation presented by William H. Millett to the faculty of the Graduate School of the University of Pennsylvania in partial fulfillment of the requirements for the degree of Doctor of Philosophy, June, 1942.

CORRECTION: The figures given in the first paragraph, page 130 of the February, 1947 Journal (Vol. 243, No. 2) of "8,000–10,000 tons of tin" should have read "800–1000 tons."

New Oscillating Bed For Polio Patients.—A new type oscillating bed being used in Pittsburgh to combat the crippling effects of polio has provided a new use for General Electric Thy-mo-trol drive, the versatile electronic system of controlling D-C motors from A-C power lines.

Built by Respir-Aid, Inc., Toledo, the new bed is mounted on a motor-driven frame which the Thy-mo-trol drive see-saws in rhythm with the patient's normal breathing. By simply turning a small knob which electronically controls the speed of the driving motor, an attendant can closely regulate the speed of the see-saw motion. A small A-C motor regulates the arc of travel of the bed's motion.

The principal function of the bed is to aid the patient in breathing. It was pointed out by Toledo Public Health officials, however, that it should not be limited to polio victims, since it has many other uses in treating respiratory and circulatory diseases. The patient may sleep in any position without losing effectiveness of the treatment. Unlike the iron lung, neither the bed nor the motion restricts usual care or treatment of the patient in any way.

The bed is to be put into production immediately, it was revealed by Lyle Burdette, president of Respir-Aid, Inc.

R. H. O.

A B-29 Turret For the Earth-Bound "Gunner."—The famed B-29 remote control gun turret which made aerial combat history in the Pacific in World War II is making amusement history in "penny" arcades throughout the country today. A thin dime in the slot will entitle the most earth-bound "gunner" to the thrills of aerial combat, manning a B-29 turret whose equipment was built by the General Electric Company for the Army Air Forces.

Stepping up to the sighting station, the intrepid gunner brings his guns to bear, by remote control, on the approaching "enemy." When he is "on" the target, the gunner depresses the trigger on the sighting station, and the silence is shattered by the "ack-ack-ack" of the .50 calibre machine guns in the turret. When the "enemy" is hit, a gong rings.

A beam of light has replaced the flying lead, and the "enemy" aircraft is a model plane embodying a photoelectric cell. The light beam from the turret guns, striking the photoelectric tube, rings a gong, signifying a hit. A hundred "rounds" can be purchased for a dime.

The sighting station contains four small selsyns, and the trigger is operated by a G-E Switchette. A small G-E lamp produces the focusing pattern. The amplidyne generators for the turret draw their power supply from a d-c generator located on the premises. All other equipment for this amusement attraction is operated from ordinary A-C city power lines.

R. H. O.

EARLY STEAM ENGINES IN AMERICA.

BY

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Fifty years ago a member of a national engineering society canvassed his historically minded colleagues for evidence of the earliest applications of steam power in America. The replies were not enlightening. When the reader surveys the collected evidence he obtains a distorted picture of the source from which our engineering heritage has sprung. As nothing appears to have been attempted since that time, it is the purpose of this paper to remedy the situation.

Let us go back into the origins of our social structure and we shall find that engineering has always been the handmaiden of civilization. The history of mankind has been a continuous quest for more power to perform work. Man raised himself from the level of the animals when he learned to apply the mechanical powers of the lever, the inclined plane, the wheel and axle, and the pulley. With each amplification of his power he gained time to think, and much of his thought was devoted to uncovering fresh means of extending this power. He gradually learned to amplify it by utilizing the natural sources of power, wind and water, to operate his machines.

The capital weakness of wind and water as sources of power does not reside in their inefficiency, especially after the invention of the Fourneyron water turbine, but in their irregularity. Dependence upon strong winds or a regular flow of water restricted the application of the windmill and the waterwheel. These narrow limitations caused man to seek some other source with a more general application. The growth of mining translated a simple desire into an imperative need.

The early colonists were continually buoyed by the hope and expectation of finding mines of rare metals anywhere in America. Each discovery was hailed as the precursor of the development of untold riches. Thus, when one of Arent Schuyler's slaves found a rich specimen of copper ore on the eastern bank of the Passaic River, it was generally accepted that an unusually valuable discovery had been made. Exaggerated notions of the value of the Schuyler mine at Belleville, N. J., lingered for a long time, long after the owners had found their mine was afflicted with the same perversity as mines in the Old World. It flooded with water beyond their powers of pumping.

Writing about it in February, 1750, Benjamin Franklin sheds a gleam of hope on the deplorable situation when he wrote: "I know of but one

valuable copper mine in this country, which is that of Schuylers in the Jerseys. This yields good copper, and has turned out vast wealth to the owners. I saw it last fall, but they were not then at work. The water had grown too hard for them, and they waited for a fire-engine from England to drain their pits. I suppose they will have them at work next; it costs them one thousand pounds sterling."*

From this letter we learn that the introduction of the steam engine into this country was due to the identical cause which had brought about its creation in England, where it replaced horses for working the pumps at flooded mines. As yet, this was the only application of steam power.

Neither Franklin nor anyone else has explained how the first steam engine was obtained. One of the contemporary owners of the mine, Colonel John Schuyler, enjoyed the favor of the royal governors of New York and New Jersey. He may have exercised his influence to have them urge the British authorities to expedite the engine, for it was four years before it arrived in this country. The manufacture of metal goods was prohibited in the colonies. British authorities forbade the export of machinery or drawings from which machines could be made, and did their best to discourage the emigration of mechanics capable of building machines. Much of the four years delay may have been devoted to efforts to have the prohibitions removed in favor of the "one valuable copper mine in the country."

Colonel Schuyler's agent in London engaged Josiah Hornblower to superintend the construction and erection of the fire-engine. Josiah was the younger brother of Jonathan Hornblower who was later to achieve eminence as the inventor of the compound engine. Both the Hornblowers rated as "engineers." In their time the term had a distinction it no longer possesses, for there was as much difference between an engineer and the manufacturer of engines as there is now between an architect and a house builder. Boulton and Watt were probably the first to combine the two branches of designing and constructing engines.

Josiah Hornblower was competent to examine the work that had to be done, estimate the capacity of the engine's parts, make what drawings were necessary, and superintend the engine's erection. He had worked under his elder brother in superintending the erection of Newcomen engines in different parts of England before he was summoned to London to receive Schuyler's offer.

In anticipation of difficulties, he had the foresight to bring with him the greater part of three steam engines. This was a wise precaution because, should replacements become necessary there was not a shop in America where the necessary castings might be made or a cylinder bored. The engine would have had to lie idle while substitute parts were shipped from England.

* Sparks, J. "Works of Franklin," Vol. VI, p. 107.

The newspapers of the day displayed a lack of enterprise in failing to mention the arrival of the ship bearing the precious cargo. One feels that such a portentous event as the introduction of steam power into the country would have justified banner headlines, but the occasion escaped all mention.

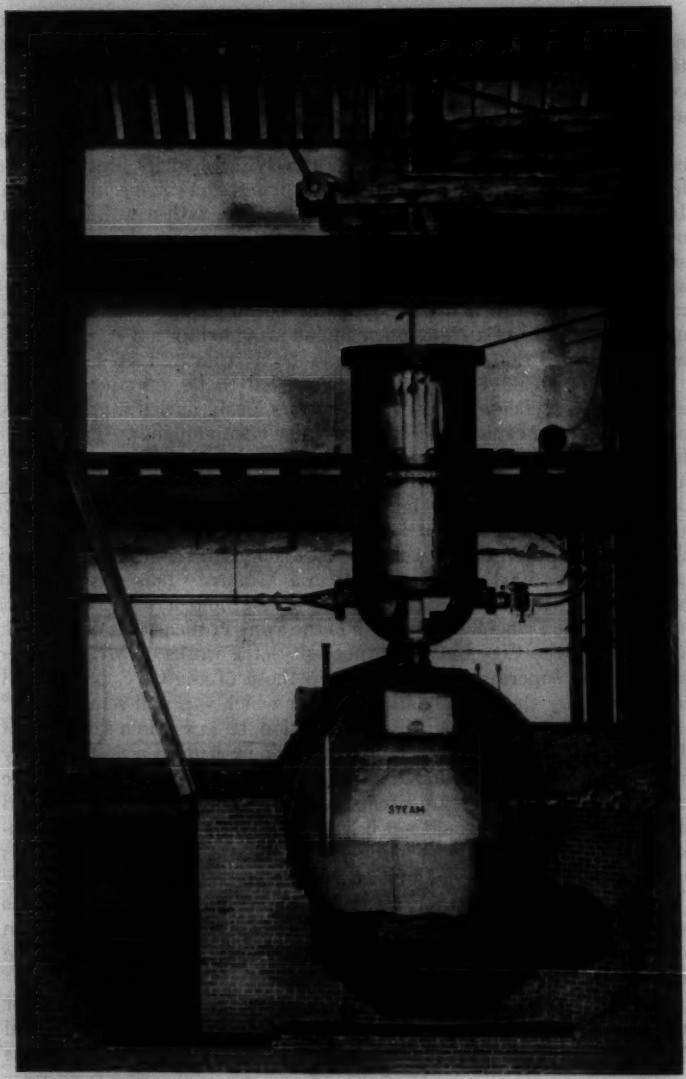
Hornblower occupied almost eithteen months in erecting the engine and in preparing it for the great day in 1753 when its fires could be lighted. Few details are known of the first steam engine in America. It was a typical Newcomen atmospheric engine with a walking beam, one end of which clutched the pump rod. A relic of the engine was shown at the Centennial Exhibition at Philadelphia in 1876. This was a cylindrical casting 4 ft. in length, with a bore of 2 ft. 10½ ins., and retaining a portion of a nozzle, probably the injection nozzle. As the usual proportions of a Newcomen engine provided for a 6 ft. drive, this would indicate that the cylinder had been truncated at some time.

The first steam engine naturally proved to be a source of much wonder to travellers, one of whom unconsciously misled a great many people upon the capacity of contemporary steam engines. Lieutenant Isaac Bangs visited the Schuyler mines on June 22, 1776 and recorded his impressions of them in simple words. "The Work which we could perceive had been done in them," he wrote, "was sufficient to astonish any Man who had seen so little of the World as I had. The Engine (for throwing off the Water) cost about three Thousand Sterling and would cast out of the Earth 80 Hogsheads in a Minute. This was actuated by Fire, and from fire it had its only Motion, and it was constructed upon the same Principles and much in the same Form as that of N. York for watering the City but (from necessity) the works of Mr. Schuyler were greatly superior in Magnitude to those of the City, of which I could judge by the incombustible Matter which was still remaining."*

Bangs did not see the engine in operation, he saw it in a sad state of disrepair. Three years earlier the engine house had been destroyed by fire and the engine had suffered so much damage that no work was done at the mine for twenty years, except surface digging. Had he witnessed the pump at work he would not have described its capacity as 80 hogsheads a minute. This was manifestly an error. As eight hogsheads was the average capacity of a Newcomen engine, the exaggeration likely arose from nothing more serious than a slip of the pen. But it has often been repeated, and the power of the first steam engine has been misrepresented upon the evidence of a man who was supposed to have seen it.

The times were too troubled to resume operations after the fire, or to obtain new parts for the damaged engine, so Hornblower occupied himself with politics until the New Jersey Copper Mine Association was

* *Proceedings of the New Jersey Historical Society*, May 30, 1855, Vol. VIII, p. 121.



NEWCOMEN'S ATMOSPHERIC ENGINE.
An engine of this type was installed at the Schuyler copper mine in 1753.

formed in 1793 to restart the mine. It is noteworthy, in view of the subsequent development of steam power in America, that the name of Nicholas Roosevelt appears among those of the sanguine capitalists who engaged in this enterprise. Hornblower chanced to be free of political office at the time, and was invited to assume the superintendence of the work. He immediately began to rebuild the Newcomen engine he had brought from England forty years earlier.

One would have liked to know more about the nature of the repairs and who performed them. It cannot have been an easy matter. The lever beams, arms, shafts of flywheels, bearings on which the flywheels were supported, and other parts of the engine were made from wood. But if the cylinder or boiler (which was made of copper) should have suffered injury in the fire and collapse of the building, Hornblower would encounter serious difficulties in executing the necessary repairs.

However, he got the engine to work again, but the new management of the mine proved unsatisfactory and Hornblower resigned his post as superintendent. An unsuccessful attempt to conduct operations with cheap unskilled German labor proved of short duration.

The old engine stood idle for several years. The next reference to it is made by an English traveller who comments: "The mines are now reopened, a steam engine erected to drain them, and the works are proceeding with spirit."* But not for long. Nelson reports that the old engine stood idle for several years "until the early part of the present century, when it was taken to pieces, and the copper boiler was broken up and carried to Philadelphia."†

The story of the first steam engine in America is well authenticated, but this cannot be said of either the second or the third. Latrobe, the earliest authority on the subject, is exceedingly vague. In making a report to the American Philosophical Society on May 20, 1803, he wrote: "Steam engines on the old construction were introduced into America about forty years ago. Two, I believe, were put up in New England before the Revolutionary War, and one (which I have seen) at the copper mine on the river Passaic. . . . All the principal parts of these engines were imported from England."‡

Where the two New England engines were located or to what purpose they were applied remains a mere matter of conjecture. They appear to have been lost in the mists of time. Latrobe thought that one had been erected in Boston and that it was still operating in 1803, but he admits he was imperfectly informed upon it. What appears more probable to us is that the second engine was that in New York mentioned by Lieutenant Bangs. It had not been operating for twenty years when Latrobe made his report.

* Finch, I. "Travels through the U.S.A. and Canada," 1833, p. 277.

† Nelson, W. "Josiah Hornblower," Newark, 1883, p. 50.

‡ *Transactions*, Vol. VI, 1809, p. 89.

In 1774, while the city of New York had a population of only 22,000, an English engineer named Christopher Colles offered the Common Council a proposal to construct water works for the city, embracing a reservoir on Manhattan Island into which water was to be pumped by means of a steam engine. The proposal having been accepted, Colles constructed his reservoir on the east side of Broadway, where Pearl and White Streets are now situated.

The Council issued bonds and paper bills to defray the cost of the undertaking, and on the reverse side of the paper money appears a picture of the engine Colles employed to operate the pump. It is a conventional Newcomen atmospheric engine, similar to that imported by Hornblower, as Banks surmised. This engine began operations in 1776, but the supply of water proved insufficient and, owing to the confused conditions prevailing in the city at the time, the enterprise was abandoned.

We have failed to ascertain what became of this engine. Its inadequacy may have condemned it, or it may simply have fallen into disrepair and the parts been stolen by those who had need of its timber and copper. There does not appear to have been any suggestion to restore it when the water supply next arose for discussion. Indeed, when it was proposed to resume the water supply in 1788, the City Council received a proposal from the Rumseian Society in Philadelphia that they should consider the installation of a steam engine designed by James Rumsey especially adapted for supplying towns with water. The Council invited the Society to submit a scheme to fulfill their requirements but nothing further was done in the matter. Rumsey had become entangled in the pursuit of a successful steamboat, and had little time to think of waterworks.

The steam engine had now caught the fancy of American inventors but, without exception, they planned only to improve water communications. This was quite comprehensible. There was as yet insufficient manufacturing to encourage the development of stationary engines to drive machinery. The new crop of inventors applied themslves diligently to the idea of using steam to propel a boat. The atmospheric engine answered the question for which it had been created, namely, to pump water, but it was inefficient and inconvenient for other purposes. It had several disadvantages. There was the still unsolved problem of transforming the up and down motion of the piston into rotary motion. The length of stroke varied, the piston did not move at regular intervals, and some contrivance had to be provided to drag the piston back again after the vacuum had caused it to fall. The dreamers of steamboats had to overcome all these difficulties.

Even after Watt had produced his more efficient engine with a separate condenser, seven years were to elapse before he achieved rotary motion with it. A number of men in America gave thought to the idea

of designing an engine that would be capable of turning paddle wheels and which would eliminate some of the disadvantages of the Newcomen engine.

When John Fitch first thought of using steam to propel a boat he was without the slightest idea how it could be done until he read a description of Newcomen's engine in Benjamin Martin's *Philosophia Britannica*. Disappointed to find that he had been preceded in the idea of using steam as a source of power, he set about applying the know-

WATT'S SUN AND PLANET ENGINE.
First brought to this country by the Manhattan Company for the New York city waterworks in 1799 or 1800.

ledge he had learned from the book. Unfortunately, he rejected the advice of Oliver Evans to employ the expansive power of steam and to design his engine to work side paddles. When Fitch began to run his boat on its regular passenger runs on the Delaware in 1790, his low pressure engine caused oars over the stern to kick the boat forward.

Rumsey worked upon an entirely different principle. He proposed to propel his boat by the reaction of a water jet on the body of water in which the vessel was floating. He does not appear to have contem-

plated using his engine in the operation of a mill or for driving machinery of any other kind.

We now arrive at the time when the Watt engine made its appearance in America. The first recorded instance occurred in 1799 or1800 when the Manhattan Company was formed for the ostensible purpose of furnishing New York with an improved water supply, but with the real intention of founding a bank. Some gesture of supplying water was made, embracing the provision of a steam engine to pump water into a reservoir on Chambers Street. The engine obtained for this purpose was an 18 h.p. Sun and Planet engine built by Boulton and Watt in England. The shaft worked three small pumps with common cranks.

The Sun and Planet was double acting and capable of pumping more water than the Newcomen engine installed by Colles, but complaints of the inadequacy of the City's water supply continued to harrass the Council. The fault resided in the reluctance of the company to lay pipes for the distribution of the water. When an extension of the facilities was forced upon the operating company they supplemented their engine with another of the same type and horse power.

An interesting feature of the first Watt engine was the replacement of the wooden boiler, probably of domestic construction, by one of sheet iron furnished by Boulton and Watt.

Thus far, the only steam engines, except those built for steamboats, operating in this country had been constructed in England. The reason for this neglect of domestic manufacture is to be found in the lack of facilities. Very few trades were practiced in this country. The great majority of the population were jacks-of-all-trades but farmers first and foremost. Men did not realize that any special talents were required for success in life, not even to become President, and so were reluctant to specialize in acquiring the skill requisite in building machinery.

This attitude was on the verge of sweeping changes. The first engineering works had been established by Nicholas Roosevelt in 1794 near Newark, N. J. He called it the Soho Works as a tribute to the genius of James Watt, whose plant in England bore the same name. An interesting account of the works as they existed in 1800 will be found in a letter quoted by Bathe.* The most noteworthy feature of this description is the rudimentary nature of the facilities possessed by the leading engineering firm in the country. The methods adopted resemble those practiced by Robinson Crusoe.

When Latrobe presented his famous report to the American Philosophical Society he mentioned a steam engine operating in the sawmill of Nicholas Roosevelt, his brother-in-law. The identification of the

* Bathe, D. and G. "Oliver Evans," Philadelphia, 1935, p. 70.

engine is somewhat confused, but there is reason to believe it was the very first engine to be built in this country which was applied to any purpose other than the propulsion of a boat.

Tradition says that Roosevelt was so fascinated by the Newcomen engine at the Schuyler mine (in which he held a financial interest after 1793) that he first made a model and, later, a small engine operating under the same principle. It may very well be this small engine that Latrobe had in mind. He says further that it "had all the improvements which had been made by the joint ingenuity of Messrs. Smallman and Straudinger (sic)." The allusion to these two men makes us pause in declaring the engine at the sawmill, which was operating in 1803, was the small Newcomen which Roosevelt had built.

When Roosevelt started the Soho Works his principal assistant was Charles Stoudinger (not Straudinger or Haudinger as Latrobe variously spells it), who had worked in the Boulton and Watt plant in England. Smallman, his associate, was also trained in England. The new Soho Works had scarcely begun operations when that impossible gentleman Chancellor Livingston (later distinguished for his association wth, Fulton in the *Clermont*) tried to sell them an idea for a steamboat. When they declined to buy it, the scheme was quite impracticable, Livingston inquired how much they would charge him for building a steam engine "large enough for a sawmill." Further correspondence resulted in an order to build a steamboat embodying the Chancellor's original ideas. From the beginning, the practical Stoudinger had a healthy professional contempt for the wealthy visionary's scheme, and would gladly have washed his hands of the whole project, but Roosevelt could not resist the temptation to be associated with such an influential personage as Chancellor Livingston. He accepted a sort of junior partnership in the enterprise and undertook to build the steamboat.

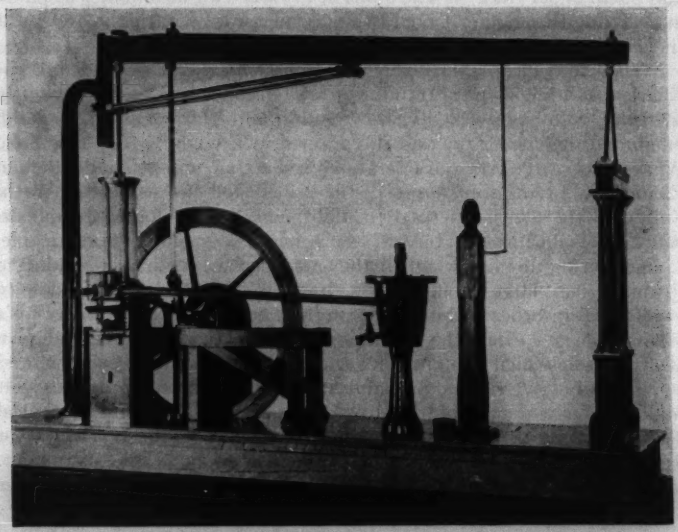
The engine was a success but the method of propulsion was an utter failure, as Stoudinger had predicted. When the futility of Livingston's idea could no longer be denied, the project was abandoned. But good steam engines were too uncommon to be cast on the scrap pile. What did Roosevelt do with it? Did he suddenly recall Livingston's original proposal to use it in a sawmill?

We cannot furnish definite answers to those questions, but we are inclined to favor the belief that it was this engine and not the small Newcomen that Latrobe had in mind when he spoke of one in Roosevelt's sawmill. He speaks only of engines of "condiderable power" and placed Evans' small engine in a separate category. A small Newcomen could hardly be classified as an engine of considerable power. Then again, Livingston's boat engine very probably had the improvements of Stoudinger and Smallman (which Latrobe says were on Roosevelt's sawmill engine).

Now, the only improvements introduced by these two English engi-

neers related to boiler design. Their improved boiler comprised a wooden chest to carry the water, in which were several upright flues connecting with the firebed. The heated air from the fire ascended these flues, presenting a larger heated surface to the water in the boiler. Wood was regarded as especially advantageous for boiler construction as it was such a poor conductor of heat.

Of what use would a boiler of this type be to a Newcomen engine? If Roosevelt modelled his Newcomen on that imported by Hornblower it would have a copper boiler. No one with experience of the two would



EVANS' "GRASSHOPPER."
The first high pressure steam engine used in this country.

ever favor the Stoudinger and Smallman boiler over Newcomen's copper boiler. As we shall see the former had a conspicuous defect.

However, the main point to emphasize is that the Roosevelt engine, whether it was his small Newcomen or the improved engine built for Livingston's boat and withdrawn from that service, appears to have been the first steam engine made in this country and employed as a stationary engine.

One project having failed, the indefatigable Livingston applied himself to another. He invented a steam engine which he claimed was vastly superior to that of Watt. He wanted Roosevelt to build it, prob-

ably at his own expense, but the engineer was too busy and too wary. He was engaged in building two large engines for the Philadelphia Waterworks, which had been ordered in the year 1800. The construction of these two engines was the most ambitious engineering feat performed up to that date in America. One of the engines had a cylinder 32 ins. in diameter with a 6 ft. stroke, and the other cylinder was 40 ins. also with 6 ft. stroke. These cylinders were cast in two pieces, which were joined with copper, and bound by a band of cast iron 18 ins. wide. When it was completed the larger of the two proved to be "at least" $\frac{5}{8}$ inch larger in diameter at the center than at the ends.

The boilers had the Stoudinger and Smallman improvements to which we have made allusion, and which were to betray their defect in these engines. Each boiler was 15 ft. long and 9 ft. square at the ends. The fire box was constructed of wrought iron and the flues of cast iron, so that the unequal contraction and expansion of the two metals caused a continual leakage until the flues were torn out to be replaced with others of wrought iron. In 1803, one of these wooden boilers was so defective that it was replaced by one made of cast iron plates, but the other lasted seven years, which was judged so satisfactory that it was withdrawn for one made of pine wood, which served until 1814.

The construction of steam engines had now gained a firm hold in the country. The period had arrived when native ingenuity had to take hold of this useful agent and give it additional power. Hitherto all the steam engines in America had employed the principle of condensing steam to create a vacuum or were low pressure double acting engines following Watt's invention. The change in principle to that in which the expansive force of the steam is utilized, was achieved by Oliver Evans who introduced the use of high pressure steam. Evans has not been given quite the full credit to which he is entitled as an inventive genius, for it was his misfortune that he independently discovered two very important applications of steam power which are usually attributed without reservation to other men. His work with high pressure steam chanced to be simultaneous with the labor of Trevethick in England, who gets most of the credit for the discovery.

Evans began to flirt with the idea of using steam in this way as early as 1773. The idea came to him during a conversation with his brother and, as this was the first major improvement in the steam engine originating in America, the story should stand alongside that of Watt watching the tea kettle boil. It is told by Evans in *Nile's Weekly Register** in these words:

"One of my brothers informing me, on a Christmas evening, that he had that day been in company with a neighboring blacksmith's boy, who, for amusement, had stopped up the touch-hole of a gun barrel, then put into it about a gill of water, and rammed down a tight wadding; after which they put the breech-end of it into a smith's fire, where it discharged itself with as

* Addenda to Vol. III, 1813.

loud a crack as if it had been loaded with gunpowder. It immediately occurred to me that there was a power capable of propelling any waggon, provided I could apply it; and I set myself to work to find out the means of doing so. I labored for sometime without success; at length a book fell into my hands, describing the old atmospheric engine. I was astonished to observe they had so far erred as to use steam only to form a vacuum, to apply the mere pressure of the atmosphere, instead of applying the elastic power of steam, for original motion; a power which I supposed was irresistible. I renewed my studies with increased ardour, and soon declared that I could make steam waggons, and endeavoured to communicate my ideas to others; but, however practicable the thing appeared to me, my object only excited the ridicule of those to whom it was known. But I persevered in my belief, and confirmed it by experiments that satisfied me of its reality."

Evans may have made some models as time permitted, for he was an exceedingly busy tradesman, but none of these have survived. Failing to receive encouragement for his idea of making "steam waggons," he determined to demonstrate the utility of the steam engine for driving machinery. In 1801 he constructed a small engine with a cylinder of 6 ins. bore and 18 ins. stroke. It was direct acting to the crank shaft and had two oscillating valves, or plug cocks, to distribute steam to both ends of the cylinder. This cost him $3700.

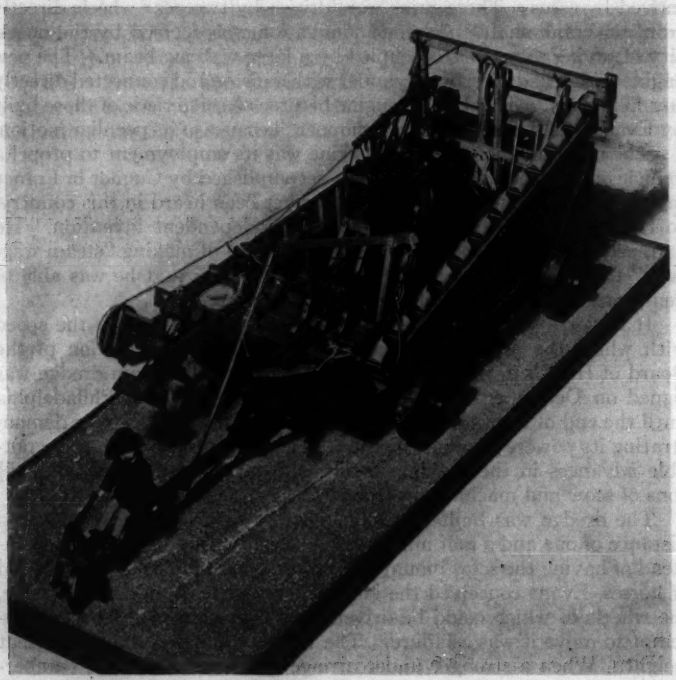
In order to give it publicity he arranged a public demonstration on Market Street in Philadelphia, where he employed the engine to drive twelve saws for cutting marble. They sawed through ten feet of marble in twelve hours. Although the demonstration attracted much attention it failed to convince the public of the significance of steam power. Accustomed to think of the waterwheel as the prime mover, the spectators could not comprehend how it could be supplanted by a small steam engine. Evans thereon withdrew his small engine from public view, and set it to work grinding plaster of paris. It was capable of grinding twelve tons in twenty-four hours.

We do not know how Evans machined up his parts. He may have had the castings done at Nancarrow's foundry at 8th and Walnut Streets. The boiler was of wood, but it departed from the conventional design in being conical and bound with iron hoops.

However, the little engine was not entirely without appreciation. A small group of men engaged Evans to build a larger engine for the purpose of propelling a boat. This had a 9 in. cylinder and a 36 in. stroke. It was completed by the end of 1802 and shipped by sea to New Orleans where it was to be installed in the hull. Unfortunately, a storm had wrecked the hull of the vessel, so that Evans was deprived of the honor of preceding Fulton in popular imagination as having been the originator of the steamboat. Instead, another honor, if somewhat dubious, was reserved for this engine.

The injured feelings of the shipping company were slightly mollified by the sale of their engine. It was installed in a sawmill at Donaldsville, La., where it speedily proved its value. The mill was able to saw 3,000 feet of boards in twelve hours. But this efficiency was the cause

of its ruin. The hand sawyers of the neighborhood acquired a higher appreciation of the steam engine than had the Philadelphians, for they could foresee the time when it would displace their labor. They were, accordingly, bent on destroying it, and on their third attempt succeeded in burning down the mill. Evans thus acquired the dubious credit for



ORUKTER AMPHIBOLOS.
Although amphibian, this was the first steam locomotive built in America.

having provoked the first technological controversy, a dispute over displaced labor.

No drawings of this engine have been preserved, but it was most probably an enlarged version of the original little engine. Its success convinced Evans that he could build steam engines of higher power than those of either Newcomen or Watt. In 1803, he advertised that he was prepared to build them for any purpose served by machinery. His en-

terprise did not promote any sales so he was led to solicit the city's Board of Health for a contract to build a steam dredge.

The engine he built for this embodied several improvements on his early engine. The cylinder had four drop valves in place of the two oscillating valves, and steam could be cut off at any predetermined point of the stroke. The valves were lifted from their seats by four levers moved by cams. These cams were driven by two gear wheels directly from the crank shaft. A second innovation was decreed by the necessity of saving some space occupied by a large walking beam. The new engine design had a vertical cylinder with a piston rod connected directly to a half beam. This type of engine became characteristic of those built by Evans, and was called a "grasshopper" because of its peculiar motion.

The main distinction of this engine was its employment to propel a vehicle on land, an achievement first accomplished by Cugnot in France in 1770. The name of Cugnot had not yet been heard in this country, so that Evans is entitled to the credit of independent invention. He had, as we have seen, been possessed of the idea of making "steam waggons" for twenty years, but it was not until 1805 that he was able to demonstrate the practicability of his conception.

It is worth while, however, to interject a comment upon the speed with which the new engine was constructed. The resolution of the Board of Health to proceed with the building of the steam dredge was signed on December 10, 1804. Evans was absent from Philadelphia until the end of February following. Yet the new dredge was demonstrating its powers in public in July. There must have been some notable advances in the building facilities to permit construction of 15½ tons of scow and machinery within this short time.

The dredge was built at Evans' shop on Market Street at 9th, a distance of one and a half miles from where it was to be launched. Instead of having the scow mounted on wheels and hauling it with the aid of horses, Evans conceived the idea of fitting it with an undercarriage, the wheels of which could be driven by the machinery. The first attempt to move it was a failure. The wheels collapsed under the great weight. When a stronger undercarriage had been made, the cumbersome amphibian proceeded under its own power to Center Square (now the City Hall Square) where, for several days, the populace of the city had the privilege of witnessing the operation of the first steam locomotive in America.

After the demonstration, the dredge, still under its own power, made its way to the Schuylkill River, where it was floated off its undercarriage, fitted with a stern wheel, and sent forth on its maritime existence. It steamed down the river to its junction with the Delaware, where it turned up the broader stream to begin its operations around Philadelphia's wharves. All progress, whether on land or water, was by steam power.

It cannot be said that the *Orukter Amphibolos*, as Evans called it, brought its inventor national fame, for the strange monster had to be seen to be believed, but it marked a turning point in the history of the steam engine in America. Had Fulton learned of Evans' work it is unlikely that he would have gone to the trouble and suffered the delays of having to obtain a Boulton and Watt engine for his *Clermont*.

Nonetheless, Evans had established a reputation of sorts. He founded the Mars Works at Ridge Avenue and 9th Street, where he built nearly one hundred steam engines during the remaining ten years of his life. From the time these works were opened, it becomes difficult to trace the history of individual steam engines in this country. Those which stand on the pedestal of time have alone engaged our attention.

Dairy Test is Sensitive.—If a cheese maker accidentally spilled as little as a pint of raw milk into a ton lot of pasteurized milk, U. S. Department of Agriculture scientists could now detect the fact and give a close estimate of the quantity of raw milk, if supplied with a sample of the Cheddar cheese made from the milk. That suggests the delicacy of the improved test announced recently by Dr. George P. Sanders of the Bureau of Dairy Industry.

What is known as the phosphatase test has been used by the dairy industry and by public health officers to detect and measure under-pasteurization of milk. The modification worked out by Dr. Sanders simplifies this and makes it much more sensitive. The test has been improved for milk and cream and can now be used to test Cheddar cheese. The Bureau is making further development of the test to apply it to other types of cheese—in fact, to all dairy products. It works well in testing goat milk and a modification of the test to apply to cheese from goat milk is an immediate aim in the laboratory work. Unpasteurized goat milk may carry Malta or undulant fever.

Extension of the phosphatase test to cheese is regarded as important because it is now generally recognized that cheese made from pasteurized milk is a safer food than raw milk cheese. The Bureau of Dairy Industry has developed a method of making high quality cheese from pasteurized milk. Pasteurization kills any disease-producing organisms that may be in the milk. The dairy industry is adopting this safer method of making cheese and several States now require that all cheese marketed in the State must be made from pasteurized milk. The new test will aid in enforcement of such legislation.

R. H. O.

Casein Bristles Produced Commercially.—The U. S. Department of Agriculture announced the successful commercial application of its development for making bristles from casein. Dr. Louis B. Howard, Chief of the Bureau of Agricultural and Industrial Chemistry, stated that research on the new product was initiated four years ago at its Eastern Regional Research Laboratory in Philadelphia to find a suitable substitute for imported natural bristle.

The artificial fiber, developed by Dr. Thomas L. McMeekin and associates, attracted the interest of numerous companies and one of these, The Rubberset Company of Newark, is formally opening a new factory at Salisbury, Maryland, for the manufacture of casein bristle and brushes using the process developed by the Department scientists.

The artificial bristle is made by extrusion of a mixture of casein and water through a suitable die, and then subjected to finisning operations. The final product is round in cross section and has a black color comparable to that of horsehair or pig bristle. Like other artificial fibers, it can be produced in any length desired and in a range of diameters. Of many possible uses, the bristle is particularly adapted to the construction of paint brushes since it is resistant to oils and organic solvents.

Casein from waste skim milk is the main constituent used in making the bristle product, but it is possible that other protein materials derived from agricultural commodities may also be employed. Although the supplies of natural bristle have improved, it is anticipated that there will be an increasing demand for high-quality, low-cost artificial bristle.

R. H. O.

THE CHARGE EFFECT IN RELATION TO THE KINETICS OF PHOTOGRAPHIC DEVELOPMENT. V. DEPENDENCE UPON GRAIN SIZE AND IODIDE CONTENT OF THE EMULSION.

BY

T. H. JAMES.

Communication No. 1125 from the Kodak Research Laboratories.

I.

The ions or molecules of the developing agent must reach the surface of a silver halide grain before they can participate in the normal direct development of that grain. This involves penetration of the double-layer field surrounding the grain. The field does not significantly influence the approach of molecules of a non-ionized developing agent (such as a member of the *p*-phenylenediamine series containing no acid group), but it acts as a barrier inhibiting the approach of *ions* of a developing agent. The preceding papers [1] of this series have dealt with modifications in development kinetics which result from this barrier effect.

The extent to which the barrier influences the kinetics of development by negatively charged ions depends both upon the charge of the ion and the magnitude of the barrier potential. The latter is not accurately known, since it is not susceptible to direct measurements by the methods usually employed in determining zeta potentials. In general, experiments on the charge effect in development have shown that additions to the developing solution of compounds or ions which would be expected to increase the barrier potential accentuate the charge effect, and additions which would be expected to decrease the potential decrease the charge effect. Only two emulsions had been used in these experiments, however, and both were motion-picture positive-type emulsions of comparable speed and average grain size. In the present investigation, emulsions which had average grain sizes varying over a large range, and which showed widely varying emulsion speeds, were employed. The purpose of the investigation was to determine to what extent the characteristics due to the charge effect varied with grain size and iodide content of the emulsion. The results show that the variations, if real, are not large.

The emulsions employed were obtained from A. P. H. Trivelli of these Laboratories, and had been prepared according to the procedure

[1] T. H. James: *J. Franklin Inst.*, 240, 15, 83, 229, 327 (1945).

given by Trivelli and Smith.[3] The emulsions were of relatively simple composition. All contained the same amount (moles) of silver halide per unit area of coated emulsion, and all emulsions had been adjusted to the same pH and pAg values for coating. The silver halide of one series of emulsions was 100 per cent. silver bromide; that of the other series was 97.44 mole per cent. silver bromide and 2.56 mole per cent. silver iodide. Table I gives the grain statistics of the undeveloped emulsions.

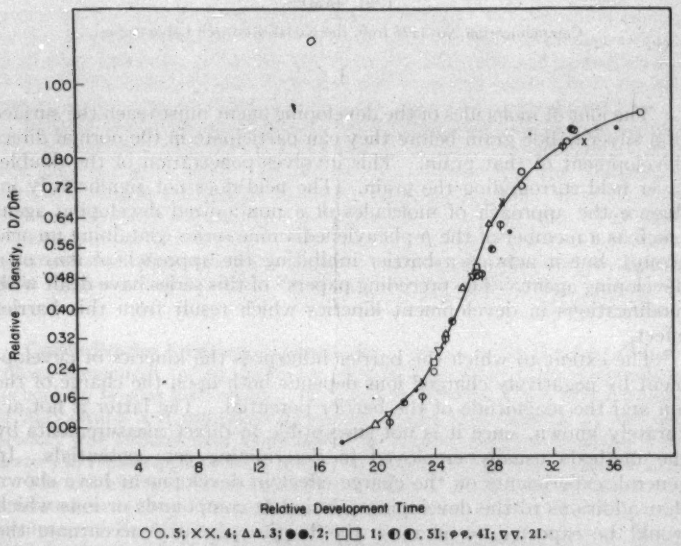


FIG. 1.

One of the outstanding manifestations of the charge effect in development is the relatively large induction period obtained upon development by doubly charged ions in the absence of sulfite. The effect is particularly marked when the developer solution contains excess bromide ions, since the bromide ions decrease the rate of development during the induction period to a considerably greater extent than they decrease the rate of subsequent development. Any variation with conditions of development in the relative extent of the induction period (as usually measured) can be detected easily by plotting density-time of development curves and adjusting the scale for each so that the

[3] A. P. H. Trivelli and W. F. Smith: *Phot. J.*, **79**, 330 (1939).

upper portions of the various curves coincide or are parallel to each other. If the same emulsion is used for each experiment, and only the conditions of development are varied, suitable adjustments in the time scale usually suffice. If different emulsions are used, however, and the comparison is made for equal development conditions, adjustment of both the time and the density scales generally becomes necessary.

Figure 1 shows (adjusted) data obtained with the emulsions listed in Table I, using a developer of the composition: Hydroquinone, 0.05 *M;* potassium bromide, 0.00167 *M;* borax-boric acid buffer to give a pH of 8.83. Sensitometer strips of the various emulsions were developed simultaneously in the absence of oxygen. The data represent densities obtained at an exposure which was well on the shoulder of the characteristic curve (but not in the solarization region), and the densities

TABLE I.

Grain Statistics of Undeveloped Emulsions.

Designation	Mole % AgI	Average Grain Size, μ^3	Dispersion of Sizes, μ^3	Average Thickness, μ	No. per cc.	Total Projection Area in $cm.^2$
1I	2.56	0.07	0.05	0.168	$477.3 \cdot 10^9$	338.88
2I	2.56	0.20	0.17	0.225	127.6	255.2
3I	2.56	0.67	0.63	0.367	23.9	155.16
4I	2.56	1.38	1.21	0.504	8.2	113.16
5I	2.56	3.77	2.87	0.800	1.89	71.25
1	—	0.11	0.09	0.234	221.46	243.61
2	—	0.48	0.68	0.22	50.08	259.58
3	—	1.76	2.70	0.243	13.34	234.78
4	—	3.70	5.76	0.289	5.34	197.58
5	—	8.53	13.48	0.395	1.69	144.16

were reduced to "comparable" values by dividing each measured density by the corresponding value for the density obtained after development to completion. Within the limits of experimental error, the points for the various emulsions fall upon a single curve, showing that no significant variation occurs in the relative induction periods, expressed on a *density* basis.

Table II gives rate and density values obtained with the hydroquinone developer used both in the presence and in the absence of added bromide. The values of Rt, where R is the maximum slope of the density-time of development curve and t is the time required to obtain a density of 0.1 $D_{max.}$, show some variations from emulsion to emulsion, but the variations are not systematic. ($D_{max.}$ is the density obtained when all of the grains of the emulsion are developed. D_∞ is the density obtained by full development of the particular exposure after correcting for fog density.)

The values of $D_{max.}$ given in Table I have an interesting implication

quite apart from our present problem. It is commonly assumed [3] that the optical density of completely developed grains is proportional to the total projection area of the undeveloped grains. The densities obtained with full development of the iodobromide emulsions are nearly proportional to the total projection areas, but this relation does not hold for the pure bromide emulsions. The failure is easily seen by comparing the density values for Emulsions 1–3. These emulsions all have substantially the same value for the total projection area, since the average thickness of the grains is nearly constant. The maximum density values, however, vary by a factor of two. The relation between density and grain statistics will be considered more fully in a subsequent publication.

TABLE II.

Rate Data for Sulfite-Free Hydroquinone Developer, pH 8.83; Log E = 1.90.

Emulsion	$D_{max.}$	No Added Br^-			0.00167*M* KBr	
		t for $0.1D_{max.}$	$100\ R/D_{max.}$	$100\ Rt/D_{max.}$	t'	$100\ R't'/D_{max.}$
1	4.4	4.0	8.6	34.4	24.2	123
2	3.8	4.4	7.5	33.0	22.2	129
3	2.2	3.6	8.0	28.8	19.8	115
4	1.9	4.4	8.3	36.5	21.1	114
5	1.3	4.0	8.1	32.4	24.5	164
1I	5.0	4.6	10.2	46.9	20.4	163
2I	4.4	4.8	8.4	40.3	19.3	137
3I	2.6	4.8	8.2	39.4	21.8	139
4I	1.7	6.1	7.9	48.2	28.9	165
5I	0.95	5.4	6.8	36.7	34.0	153

The preceding rate data are for equal exposures of all emulsions used. The emulsions, however, vary in maximum speed by about eightyfold. Any comparison of rates of development of photographically useful exposures must take this into consideration. Table III gives data for exposures chosen in such a way that, when development is carried to completion, the density obtained is three fourths that of the $D_{max.}$ for the emulsion. These exposures give a degree of developability which, on a density basis, is equal for all emulsions. The table also contains data for a constant exposure which, for most of the emulsions, lies in the photographically useful range.

Both for the constant exposure and for the exposure adjusted to give the same value of $D_\infty/D_{max.}$, the duration of the induction period increases with increasing grain size in each emulsion series. Entirely similar results were obtained with a ferro-oxalate developer.

Variations in the induction period of the grains from emulsion to

[3] Cf. L. Silberstein and A. P. H. Trivelli: *J. Optical Soc. Am.*, **28**, 441 (1938). J. H. Webb: *Ibid.*, **29**, 314 (1939).

TABLE III.

Emulsion	log E = 0.25		Exposure to Give D_∞ = (3/4) $D_{max.}$	
	t for 0.1 D_∞ (no KBr)	100 R/D_∞	t for 0.1 D_∞ (no KBr)	t for 0.1 D_∞ (0.00167 M KBr)
1	5.7	5.6	5.6	34
2	7.1	7.0	6.2	34
3	8.7	5.5	9.5	40*
4	10.5	5.0	10.4	50*
5	12.0	5.8	11.0	70
1I	6.7	7.0	5.1	—
2I	6.1	—	5.3	28
3I	6.3	7.4	6.6	35*
4I	7.6	5.6	8.7	49
5I	8.0	6.8	9.8	66

* Values so marked are approximate.

emulsion conceivably could arise as a consequence of variations in the total amount of grain surface per unit area. The emulsions were all coated at the same pAg, but the decrease in total grain surface accompanying increase in grain size might result in an effectively greater concentration of *adsorbed* bromide ions in the large grain emulsions than in the small grain ones. It can be shown readily that a change in total available surface does not significantly alter the induction period, provided the grain size does not change. A series of emulsions was prepared by diluting the standard emulsion No. 2 with gelatin. All emulsions were coated to a constant thickness. The results of development rate determinations on these emulsions are given in Table IV. No significant change of rate with total area occurs.

TABLE IV.

Development Rates for Various Dilutions of Emulsion No. 2. Standard Hydroquinone Developer Used.

AgBr as % of Standard Emulsion	t for 0.1 D_∞ D_∞ = 3/4 $D_{max.}$	t for 0.1 $D_{max.}$ Log E = 1.9
100	6.1	3.7
50	5.8	3.7
25	5.7	3.5
12.5	6.0	3.5

The data in Table III seem to show that, for constant moderate exposure or for exposure chosen to make the same percentage of grains developable in each emulsion, the induction period *increases* with grain size. The indications are erroneous, however, and serve to illustrate the type of error which can arise from the unqualified assumption that density changes in one emulsion are directly equivalent to the same density changes in another emulsion. If we had to deal with the extreme condition in which the induction period of the individual grains in a given emulsion varied markedly, and the average period was much

greater than the duration of development of a grain beyond the induction period, then an increase in density would be equivalent to an increase in the number of *developed* grains. Since in that event density is proportional to number, little error would arise from the assumption that equality of the ratio D/D_∞ for a series of emulsions means equality of the degree of development. If, however, the increase in density results primarily from an increase in the *size* of the developing grains,[1] the preceding assumption can be in great error. For the exposures under discussion (to give $D_\infty = 3/4\ D_{max.}$) the increase in density involves both an increase in number and an increase in size, but the latter is the more important factor.

If the density increase is entirely a matter of increase in size of the developing grains, and further, if the average number of development centers per developing grain is equal for all emulsions employed, an equal degree of development for each emulsion is obtained when D_1N/N' is the same for all. In this expression, D_1 is an arbitrarily chosen, sufficiently small density for an emulsion containing N' grains per unit area, and N is the number of grains in the emulsion to be compared with the first. Fulfillment of this condition implies that the grains of each emulsion have developed into silver specks of equal average size.

When the exposure is such that $D_\infty = 3/4\ D_{max.}$, the number of development centers per grain is quite small, and a more reliable comparison of rates is obtained by using equality of D_1N/N' as a criterion of equality of development than by using equality of D/D_∞. Table V

TABLE V.

Comparison of Emulsions for $D_\infty = 3/4\ D_{max.}$

Emulsion	1I	1	2I	2	3I	4I	3	4
t_e	9.0	7.3	6.0	5.0	5.2	6.3	6.5	6

gives values of t_e, representing the time in minutes required to obtain the density D_1N/N', where D_1 was chosen as $0.3\ D_\infty$ for Emulsion No. 1. With the exception of the two emulsions of smallest grain size, t_e shows little or no genuine dependence upon grain size.

At the high exposure used to obtain the data given in Table II, an increase in density with time of development results primarily from an increase in size of the developed silver particles. The validity of the assumption that the number of development centers per grain is nearly independent of the size of the grain is very doubtful, however, and calculation of rate values on the basis of this assumption leads to contradictory results. Table VI gives results of such a calculation. The t_e values have been determined as before. The R values were determined directly from the slope of the density-time of development

curve at a point on the curve corresponding to the density, D_2N/N', where D_2 was chosen as $0.5D_\infty$ for Emulsion No. 1. The fourth column of Table VI gives the slope rates reduced to equivalent rates (on the basis of the preceding assumptions) by dividing them by N.

If the average number of development centers per grain is the same for all emulsions at this high exposure, the size of the development

TABLE VI.

Comparable Rate Data Calculated on Assumption of Parallel Development of All Grains. Log E = 1.9.

Emulsion	t_e	R	$\frac{R \times 10^{11}}{N}$	$\frac{R \cdot t \cdot 10^{11}}{N}$
1I	11.0	0.50	1.05	11.5
1	7.0	0.40	1.8	12.6
2I	5.5	0.35	2.8	15.4
2	4.0	0.21	4.2	16.8
3I	3.5	0.10	4.2	14.7
3	2.7	0.08	6.0	16.2
4	2.0	—	—	—

centers should increase with increasing grain size, since the larger grains absorb more light per grain than do the small ones. The induction period, accordingly, should be smaller. This is in agreement with the results given in Column 2. In fact, a smooth, nearly linear curve is obtained by making a log-log plot of t_e against average grain size. However, the reduced values, R/N, should then be constant, since the rate of development beyond the induction period, on this basis, should not depend upon the exposure. Actually, the calculated rates show a marked increase with grain size.

II.

It was shown in Part III of this series [1] that gamma values considerably greater than γ_∞* could be obtained by developing the emulsions used in certain sufite-free hydroquinone solutions containing potassium bromide. The typical course of development is that gamma increases rapidly during the early stages, passes through a maximum, and subsequently decreases. Certain commercial high-speed emulsions failed to show this course, but these emulsions were all color-sensitized, and it was found that the high-gamma effect is decreased or not obtained at all if the motion-picture positive emulsions are treated with a cyanine sensitizing dye before development. It remains to be established, however, that the high-gamma effect can be obtained with relatively

* γ_∞ is the gamma value obtained by full development of the emulsion, using fog-corrected density values. Usually, $\gamma_{max.}$, the maximum value obtained experimentally, is equal to or smaller than γ_∞.

high-speed, non-optically sensitized emulsions of wide grain-size distribution.

Table VII gives data obtained with the two series of emulsions used in the present work. The hydroquinone developer contained 0.05*M* developing agent, no sulfite, and was buffered at pH 8.83. Sensitometer strips were developed both in the presence and absence of added KBr. Even in the presence of bromide, this developer did not give the very short toe obtained previously with the high-pH hydroquinone developer, but it gave gamma values considerably greater than γ_∞.

TABLE VII.

Development in Hydroquinone Solution of pH = 8.83.

Emulsion	(no added KBr) $\gamma_{max.} = \gamma_\infty$	(0.00167*M* KBr)		Ratio, $\gamma_{max.}/\gamma_\infty$
		$\gamma_{max.}$	γ_∞	
1	2.4	2.6	5.3	2.05
2	2.1	2.2	4.5	2.05
3	1.53	1.5	3.5	2.33
4	1.33	1.2	2.6	2.17
5	0.97	0.93	1.90	2.04
1I	2.35	—	4.6	(1.92)
2I	1.72	1.98	2.84	1.43
3I	1.14	1.35	1.50	1.11
4I	1.00	0.94	1.20	1.10
5I	—	0.70	0.70	1.00

In the absence of added bromide ion, the hydroquinone solution yields $\gamma_{max.}$ values which are practically identical with the γ_∞ values. In the presence of excess bromide ion, however, the $\gamma_{max.}$ values are greater in most cases (Table VII). The $\gamma_{max.}$ values are approximately twice the γ_∞ values for all of the pure bromide emulsions. The ratio of $\gamma_{max.}$ to γ_∞ decreases in the iodobromide series as grain size (and emulsion speed) increases. Table VIII, however, shows that even these emulsions yield markedly increased $\gamma_{max.}$ values when development is carried out in the high-pH hydroquinone solution (hydroquinone, 0.005*M*, KBr, 0.00667*M;* pH = 12.7). Thus, under proper conditions, it is possible to obtain gamma values double those of γ_∞, using emulsions varying eightyfold in speed, 120–fold in average projection area,

TABLE VIII.

Development in Hydroquinone Solution at pH = 12.7 (0.00667 M KBr).

Emulsion	γ_∞	$\gamma_{max.}$	Ratio $\gamma_{max.}/\gamma_\infty$
1	2.40	5.4	2.25
2I	2.05	4.15	2.02
3I	1.33	3.1	2.33
4I	0.95	2.04	2.15

TABLE IX.

Development of Dilute Emulsions in Sulfite-Free Hydroquinone at pH = 8.83 (0.00167 M KBr) and pH = 12.7 (0.00667 M KBr).

AgBr as % of Standard Emulsion	pH	γ_∞	$\gamma_{max.}$	Ratio
100	8.83	2.5	5.3	2.12
50	8.83	1.43	3.1	2.16
25	8.83	0.77	1.7	2.19
12.5	8.83	0.37	0.85	2.29
100	12.7	2.3	5.4	2.25
50	12.7	1.4	3.3	2.36
12.5	12.7	0.4	1.08	2.7

and made up of either plain silver bromide grains or grains containing 2.56 per cent. iodide.

The exceptionally high gammas obtained in the early stages of development are a consequence of the action of an oxidation product of the hydroquinone. The oxidation product produced at the grains which first start to develop spreads to other grains which are developable, but possess longer induction periods. An action of the oxidation product shortens these induction periods and thus increases the rate of formation of density. The maximum distance at which a grain can be affected in this way is not known, but the grain certainly need not be

TABLE X.

Effect of Bromide-Ion Concentration upon Development by Sulfite-Free Hydroquinone at pH = 8.83; log E = 2.05.

Emulsion	KBr	t for 0.1 $D_{max.}$	R	Rt
2	0.00000	4.75	0.110	0.522
	0.00033	13.8	0.089	1.23
	0.00100	23.4	0.088	2.06
4I	0.00000	7.7	0.064	0.493
	0.00033	24.1	0.057	1.37
	0.00100	38.5	0.057	2.17

in close contact with the initially developing grain. This is shown by some experiments on diluted emulsions. Table IX gives results obtained with the standard emulsion, No. 1, diluted to various degrees before coating. The ratio of $\gamma_{max.}/\gamma_\infty$ is just as large for emulsions containing only one-eighth as much silver bromide per unit area as the standard.

Finally, comparisons were made of the effect of bromide ion and phenosafranine upon the development rates of two emulsions which differed considerably in grain size and composition, but had almost identical total surface per unit area. The emulsions used were 4I of normal coating strength and 2 diluted to 50 per cent. of the normal silver

bromide per unit area of coated emulsions. When the total surface area of the silver halide was calculated on the basis of the assumption that the grains can be treated as flat cylinders, the areas obtained were 397 and 385 cm.^{2} per cubic centimeter or emulsion, respectively. The

TABLE XI.

Effect of Phenosafranine upon Development by Hydroquinone Solution Containing 0.00033M KBr, pH = 8.83; log E = 2.05.

Emulsion	Phenosafranine	t for 0.1 $D_{max.}$	R	Rt
2	0.000000	13.8	0.089	1.23
	0.000688	9.7	0.173	1.68
	0.00344	3.9	0.318	1.24
	0.0172	2.7	0.348	0.94
4I	0.000000	24.1	0.064	1.37
	0.000688	14.3	0.099	1.42
	0.00344	5.2	0.194	1.01
	0.0172	2.1	0.202	0.41

$D_{max.}$ values were 1.7 and 1.65. Tables X and XI give rate data for various concentrations of bromide ion and phenosafranine.

These data show no evidence of qualitative differences in the behavior of the two emulsions. Quantitative differences are present, but even these are small, and the general trend of the data is the same for both emulsions.

Rochester 4, New York.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

STANDARDS FOR ANALYTICAL FILTER PAPERS.

Standards of quality for analytical filter papers, previously nonexistent for this important item in chemical research and analysis, are being developed by the Paper Laboratory of the National Bureau of Standards through a critical evaluation of all currently available brands. Based on improved testing methods perfected at the Bureau, together with established procedure for certain basic determinations, it is believed the standards finally recommended will provide a ready means of determining the suitability of a particular paper for its intended use. Requirements will be included for the three classes of general analytical work involving coarse, medium, and fine precipitates.

Analytical filter papers, like many other laboratory materials, have an importance to science and industry far in excess of their dollar value. Many industrial processes require quick, accurate analysis for their control and these depend, in many cases, on the speed, retentiveness, and purity of the filter paper used in making the analyses.

Filter papers have been used and bought by brand names, practically as a tradition among laboratory workers, and preference has usually been given to imported papers, most of which have come from England, Germany, and Sweden. When the war made supplies from these sources uncertain, American manufacturers began production of grades of analytical papers that had not previously been made in this country. In order to provide data that would be useful as a guide to manufacturers and users in comparing the quality of different papers, as well as to eliminate dependence on brand names, the Bureau undertook an investigation of the physical and chemical properties of the papers.

Improved methods were developed for determination of rate of flow of water through the papers, retention of fine precipitates, and ash contents, as well as a method for measuring the bursting strength of wet paper. Density, purity of cellulose, and acidity, for which definite testing procedures were already available, are also important determinations.

The method for rate of flow of water is of special interest. Unique in that it does not require the use of special, complicated apparatus, it possesses the advantage of simulating the manner in which filtrations are made. The filter paper is folded in a cone in the usual way, taking care to expel air pockets. Instead of placing in a funnel, however, the

* Communicated by the Director.

cone is suspended freely in a wire loop, thus eliminating the variables such as design, angle, and stem dimensions of the funnel. It was found necessary to prefilter the water used in the test, but when this was done, highly reproducible results were obtained.

The investigation was also extended to include a study of "shelf life" of filter papers. It was found that some papers not only became brittle with age, but that decomposition products were formed that might contaminate the solutions being filtered. An accelerated aging test previously developed for paper generally enabled a satisfactory prediction of stability to be made.

This work has shown significant differences among filter papers which, if studied critically, may point the way to future improvements. For example, some papers of equal retentativeness differ in speed, and hence may yield a clue as to how still faster papers may be made.

Measurements of the quality of three foreign and two domestic brands of analytical filter paper have shown that the American-made papers are fully equal to the imported papers. It is anticipated that the application of quality standards based on quantitative measurements will eliminate traditional buying practices and permit free competition among all suppliers.

THE FRANKLIN INSTITUTE.

MINUTES OF THE FRANKLIN INSTITUTE STATED MONTHLY MEETING IN THE LECTURE HALL, HELD WEDNESDAY, FEBRUARY 19, 1947, AT 8:15 P.M.

The Franklin Institute held its stated monthly meeting in the Lecture Hall on Wednesday evening, February 19, 1947. Mr. Richard T. Nalle presided. There were approximately three hundred people present.

Minutes of the stated monthly meeting for December had been printed in full in the January issue of the Journal and were therefore not read. The President stated that if there were no corrections or additions, the minutes would stand approved as printed. There was no contrary motion.

The President then called upon the Secretary to give the report of new admissions to the membership of The Franklin Institute.

The following members were elected during the month of January.

Sustaining	2
Active	60
Associate	28
Student	46
Total	136

The President then introduced Captain Charles Bittinger, U.S.N.R., Navy Department, Washington, D. C., who gave a very popular report on "Naval Camouflage." Captain Bittinger showed Kodachrome motion pictures to illustrate camouflage developments and the difference between the results of the atomic bomb explosions over Nagasaki and Bikini. He spoke also on camouflage of the future as modified by radio-controlled missiles, rockets, and atomic bombs. Captain Bittinger explained methods of camouflage asked him by various people in the audience.

The meeting was adjourned by the President with a rising vote of thanks to the speaker.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, February 12, 1947.)

HALL OF THE COMMITTEE,
PHILADELPHIA, FEBRUARY 12, 1947.

MR. WALTER C. WAGNER in the Chair.

The following reports were presented for final action:

No. 3169: Underwater Navigation and Listening.

This report recommended the award of an Edward Longstreth Medal to Harold John Warren Fay, of Westboro, Massachusetts, "In recognition of his foresight, devotion, scientific acumen and practical outlook in the initiation and development of the art of underwater signalling and listening."

No. 3175: Brown Medal.

This report recommended the award of a Frank P. Brown Medal to Karl P. Billner, of Philadelphia, Pennsylvania, "In consideration of his invention of a practical process for treating

concrete, together with the development of suitable equipment whereby the qualities of the concrete are improved and the elapsed time of construction is reduced."

No. 3177: Newcomen Medal.

This report recommended the award of the Newcomen Medal to Everett G. Ackart, of Wilmington, Delaware, "In consideration of his long and successful career of forty years, in designing industrial steam plants and his outstanding accomplishment in designing and building a number of these for the manufacture of smokeless powder at a time when no such plants existed and when the national emergency was great, and in the light of his great accomplishment in building, in time of peace, other installations for the generation of power and steam for industrial use."

(Abstract of Proceedings of Special Meeting held Tuesday, February 18, 1947.)

No. 3182: Clark Medal.

This report recommended the award of the Walton Clark Medal to Edward George Boyer, of Philadelphia, Pennsylvania, "In consideration of his notable improvements in the development of processes for reforming hydrocarbon gases and for his improved and simplified methods for calculating the required composition of interchangeable gas mixtures and for his other valuable contributions to the gas art."

No. 3183: Ballantine Medal.

This report recommended the award of the Stuart Ballantine Medal to George Clark Southworth, of Red Bank, New Jersey, "In consideration of his pioneer work in electromagnetic microwave technique, a material contribution to the development of new systems of communication and reconnaissance radar."

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M., Wednesdays and Thursdays from 2 P.M. until 10 P.M.

RECENT ADDITIONS.

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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

The Blood Picture of Normal Laboratory Animals.[1] A Review of the Literature 1936-1946.—MARY VIRGINIA GARDNER.

THE RABBIT.

Method of Obtaining Blood. Blood, sufficient for routine hematological examination, usually can be obtained by incising the marginal ear vein of the rabbit; larger quantities of blood can be procured by heart puncture. Small amounts of blood may be taken from rabbits frequently without danger of anemia. Removal of about 1,000 cc. of blood from a rabbit over a period of 3 months was found by Sjövall (1) to result in a decrease in the hemoglobin concentration and in the red blood cell count, and an increase in red blood cell diameter; the rabbits, however, developed normally and showed no inanition.

Red Blood Cells. The erythrocytes of the rabbit, like those of most mammals, are non-nucleated, biconcave discs which appear orange in Wright-stained preparations. Sjövall (1) found the mean diameter of the red cells in rabbits 6 months of age to be 6.9μ. The average of the reported values for erythrocyte numbers in adult rabbit blood, regardless of sex or strain, is 5,610,000 per cmm. (Table I).

The blood of new-born rabbits was shown by Sabin, Miller, Smithburn, Thomas and Hummel (2) to contain an average of 4,016,000 red blood cells per cmm. Although the number of erythrocytes was found to decrease during the first week of life, it then rose rapidly and by the 3rd week of life had reached a mean level approximating that of the adult rabbit.

Differences in the numbers of erythrocytes between strains of rabbits were found by Casey, Rosahn, Hu and Pearce (3); little variation in erythrocyte number between sexes was reported by Wintrobe, Shumacker, and Schmidt (12).

The average erythrocyte count was found by Cole, Allison, Murray, Boyden, Anderson and Leathem (4) to have decreased from 5,710,000 to 5,390,000 cells per cmm. in rabbits which had been starved for 24 hours.

A seasonal change in the erythrocyte count in male rabbits of different breeds was found, by Casey, Rosahn, Hu and Pearce (3), to amount to a decrease of 236,000 cells from spring to fall.

An average of 0.98 nucleated red blood cells per 100 erythrocytes was reported by Smith (11) in a study of 155 counts on 24 normal adult rabbits.

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[1] Continued from page 176, Vol. 243, No. 2.

Reticulocytes were found by Sjövall (1) to make up 2 to 3 per cent. of the total number of erythrocytes in the blood of normal rabbits, and by Rapoport, Guest and Wing (9) to average 2.2 per cent. of the erythrocytes in the blood of 5 New Zealand white rabbits.

TABLE I.

Erythrocytes.

Age of Rabbits	Sex and No. of Rabbits	Strain	Erythrocytes × 10^6/cmm.		Reported by
			Average	Range	
new-born	24	some New Zealand red	4.016	2.76–5.21	Sabin (2)
1 week	23	some New Zealand red	3.766		Sabin (2)
3 weeks	30	some New Zealand red	5.399		Sabin (2)
adult	24 M	Havana	5.73		Casey (3)
adult	19 M	Himalayan	5.65		Casey (3)
adult	14 M	Belgian	5.32		Casey (3)
adult	24 M	English	5.34		Casey (3)
adult	16 M	Polish	5.62		Casey (3)
adult	18 M	Dutch	5.26		Casey (3)
adult	18 M	Beveren	5.20		Casey (3)
adult	10 M	Chinchilla	4.97		Casey (3)
adult	13 M	Rex	4.87		Casey (3)
adult	8 M	French Silver	4.98		Casey (3)
adult	6 M	American Blue	5.20		Casey (3)
adult	4 M	New Zealand	5.62		Casey (3)
adult	3 M	Flemish	5.55		Casey (3)
adult	2 M	Gouda	5.98		Casey (3)
adult	1 M	Tan	5.55		Casey (3)
adult	26	domestic	5.71	4.6–6.9	Cole (4)
adult	10 M	New Zealand white	5.606		DeCourcy (5)
adult	16	—	5.67		Dougherty (6)
adult	6 M	New Zealand white and Champagne d' Argent	5.52	4.8–6.5	Gregg (7)
adult	4	—	5.75		Hueper (8)
adult	5	New Zealand white	6.696		Rapoport (9)
adult	62	New Zealand red	5.366		Sabin (2)
adult	4	albino	6.15		Sharpe (10)
adult	—	—		5.0–5.5	Sjövall (1)
adult	24	—	5.997		Smith (11)
adult	35 M	—	6.25		Wintrobe (12)
adult	26 F	—	6.30		Wintrobe (12)
Average			5.610		

Hemoglobin. The blood of new-born rabbits was found by Sabin, Miller, Smithburn, Thomas and Hummel (2) to contain 12.8 g. hemoglobin per 100 cc. The average of the reported amounts of hemoglobin in the blood of adult rabbits, regardless of sex or strain, is 12.07 g. per 100 cc. (Table II). Although there appears to be little difference in the hemoglobin content of the blood of rabbits at different ages, Sabin, Miller, Smithburn, Thomas and Hummel (2) showed that the amount of hemoglobin per red cell was high during fetal life and only gradually

approached the standard of the adult rabbit which was reached about the 3rd or 4th month of life.

A slightly higher hemoglobin level was found in adult female rabbits than in adult males by Wintrobe, Shumacker and Schmidt (12), but no definite conclusions of sex difference in hemoglobin content were drawn by them.

White Blood Cells. The leukocyte occurring in the greatest numbers

TABLE II.

Hemoglobin.

Age of Rabbits	Sex and No. of Rabbits	Strain	Hemoglobin g./100 cc.		Method	Reported by
			Average	Range		
new-born	24	some New Zealand red	12.84		Newcomer	Sabin (2)
1 week	23	some New Zealand red	11.5		Newcomer	Sabin (2)
3 weeks	30	some New Zealand red	12.51		Newcomer	Sabin (2)
adult	24 M	Havana	12.4		Newcomer	Casey (3)
adult	19 M	Himalayan	11.4		Newcomer	Casey (3)
adult	14 M	Belgian	11.6		Newcomer	Casey (3)
adult	24 M	English	11.5		Newcomer	Casey (3)
adult	16 M	Polish	12.5		Newcomer	Casey (3)
adult	18 M	Dutch	12.1		Newcomer	Casey (3)
adult	18 M	Beveren	11.5		Newcomer	Casey (3)
adult	10 M	Chinchilla	10.5		Newcomer	Casey (3)
adult	13 M	Rex	10.6		Newcomer	Casey (3)
adult	8 M	French Silver	10.9		Newcomer	Casey (3)
adult	6 M	American Blue	11.3		Newcomer	Casey (3)
adult	4 M	New Zealand	11.8		Newcomer	Casey (3)
adult	3 M	Flemish	11.7		Newcomer	Casey (3)
adult	2 M	Gouda	12.6		Newcomer	Casey (3)
adult	1 M	Tan	12.9		Newcomer	Casey (3)
adult	16	—	12.0		alkaline hematin	Dougherty (6)
adult	6 M	New Zealand white and Champagne d'-Argent	12.0		Haden-Hausser	Gregg (7)
adult	4	—	11.9		—	Hueper (8)
6 months	29	New Zealand red	13.35		Newcomer	Sabin (2)
adult	4	albino	14.7		Sahli	Sharpe (10)
6–9 months	35 M	—	12.9		Newcomer-V.S.	Wintrobe (12)
6–9 months	26 F	—	13.4		Newcomer-V.S.	Wintrobe (12)
Average			12.07			

in the blood of the normal adult rabbit is the pseudo-eosinophil. This cell is probably analogous to the neutrophil of human blood. It has the coiled, twisted or lobulated nucleus with a fine network of chromatin, typical of the cells of the granulocyte series. The cell is named "pseudo-eosinophil" because the numerous granules which fill the cytoplasm are acidophilic, and appear red or pink, in Wright-stained preparations. For this reason it may be confused with the eosinophil, the granules of which are also acidophilic. The granules of the pseudo-

eosinophil, however, are small and of uniform size, while those of the eosinophil are 3 or 4 times larger and usually cover the cell, frequently obscuring the nucleus. The basophil of rabbit blood has the typical small, dense, blue-black granules and palely basophilic nucleus in Wright-stained preparations.

The monocyte is the largest cell to be found in normal rabbit blood; the nucleus is large, eccentrically located and sometimes indented, while the cytoplasm is weakly basophilic and without granulation. The lymphocyte, the smallest leukocyte, has a large, deeply staining nucleus and a narrow rim of basophilic protoplasm. Many investigators believe it impossible to distinguish between lymphocytes and monocytes in rabbit blood without the use of supravital staining. These workers frequently distinguish the two types of cells only as large and small "mononuclears."

The averages of the reported values for the different leukocytes of rabbit blood, expressed as the number per 100 white blood cells, are:

Pseudo-eosinophils (neutrophils)	45.03	(31.7–59.3)
Eosinophils	1.62	(0.7–4.3)
Basophils	6.28	(2.4–9.0)
Monocytes (large mononuclears)	9.54	(1.3–15.8)
Lymphocytes (small mononuclears)	38.45	(19.8–68.2)

The average of the reported numbers of leukocytes found in the blood of the adult rabbit, regardless of sex or strain, is 8,600 per cmm. (Table III). The blood of the new-born rabbit was found, by Sabin, Miller, Smithburn, Thomas and Hummel (2), to contain an average of 2,525 white cells per cmm. They found that the number of white cells in the blood increased slowly, reaching the number characteristic of the adult animal only after 5 or 6 months. They found, too, that the monocyte was the first cell in the blood to reach the normal adult quota, since it appeared at the end of the first week of post-natal life in numbers characteristic of the adult.

No significant digestive leukocytosis and no significant variation in the hourly mean leukocyte counts of 204 young adult male rabbits were found by Casey (13). A 24-hour period of starvation was shown by Cole, Allison, Murray, Boyden, Anderson and Leathem (4) to lower the average white blood cell count from 8,900 to 8,340 cells per cmm. A delayed leukopenia of 14.3 per cent. was found by Nice and Katz (15) to occur after rabbits had become excited.

Platelets. The platelets of the rabbit are small, round bodies, with a colorless periphery and a clump of purple granules in the center in Wright-stained preparations. They are very numerous in early life, according to Sabin, Miller, Smithburn, Thomas and Hummel (2), who found an average of 481,000 per cmm. in the blood of the new-born rabbit. The average of the values reported in the literature (Table IV)

TABLE III.

Leukocytes.

Age of Rabbits	Sex and No. of Rabbits	Strain	Leukocytes per cmm.		Cells per 100 Leukocytes						Reported by
			Average	Range	Pseudo-eo.	Eo.	Baso.	Lymph.	Mono.	Other	
new-born	24	some New Zealand red	2,525	300–7,000	71.2	0.6	4.7	13.1	8.3	1.7	Sabin (2)
1 week	23	some New Zealand red	2,840		51.8	1.4	5.1	19.0	21.7	0.6	Sabin (2)
3 weeks	30	some New Zealand red	3,943		41.1	0.4	8.1	30.4	18.6	0.9	Sabin (2)
adult	326 M	variety	7,882								Casey (13)
adult	24 M	Havana	6,530		53.7	1.1	5.6	31.0	8.5		Casey (3)
adult	19 M	Himalayan	7,070		46.2	1.7	6.6	36.5	9.3		Casey (3)
adult	14 M	Belgian	7,760		48.9	1.9	9.0	30.6	9.7		Casey (3)
adult	24 M	English	6,800		54.1	1.7	8.5	26.7	8.9		Casey (3)
adult	16 M	Polish	8,650		43.5	1.1	3.8	43.5	8.1		Casey (3)
adult	18 M	Dutch	7,330		53.6	1.6	5.3	29.9	9.6		Casey (3)
adult	18 M	Beveren	8,580		43.8	1.7	8.5	35.6	10.1		Casey (3)
adult	10 M	Chinchilla	9,050		47.6	1.7	6.2	36.1	8.4		Casey (3)
adult	13 M	Rex	7,820		48.1	0.9	7.9	33.9	9.2		Casey (3)
adult	8 M	French Silver	8,160		47.8	0.7	4.6	32.3	14.6		Casey (3)
adult	6 M	American Blue	8,410		55.1	1.5	8.0	24.0	11.6		Casey (3)
adult	4 M	New Zealand	9,890		43.4	1.2	6.4	37.5	11.5		Casey (3)
adult	3 M	Flemish	8,020		46.5	3.4	5.9	33.6	10.6		Casey (3)
adult	2 M	Gouda	9,360		59.3	4.3	5.2	19.8	11.6		Casey (3)
adult	1 M	Tan	6,480		43.5	1.6	4.5	45.7	4.8		Casey (3)
adult	29	domestic	8,900	5,200–21,400							Cole (4)
adult	10 M	New Zealand white	9,866								DeCourcy (5)
adult	18	—	10,053		31.7			68.2			Dougherty (6)
adult	2 M	New Zealand white and Champagne d'-Argent		6,200–10,200	32.2	1.1	2.4	63.1	1.3		Gregg (7)
adult	4	—	8,100								Hueper (8)
adult	12	—	9,300								Nettleship (14)
adult	37	—	10,390								Nice (15)
adult	62	—	8,866		42.03	1.24	8.06	40.54	8.10		Sabin (2)
adult	24	—	11,797	5,575–20,775	33.5	0.8	6.6	43.0	15.8		Smith (11)
adult	11 F	—	9,900	5,000–20,000							Wilson (16)
adult	14	New Zealand white and mixed	9,449		31			62			Reifenstein (17)
Average			8,600		45.03	1.62	6.28	38.45	9.54		

TABLE IV.
Platelets.

Age of Rabbits	Sex and No. of Rabbits	Strain	Platelets × 10^4/cmm.		Reported by
			Average	Range	
new-born	24	some New Zealand red	481		Sabin (2)
1 week	23	some New Zealand red	377		Sabin (2)
adult	24	Havana	562		Casey (3)
adult	19	Himalayan	476		Casey (3)
adult	14	Belgian	527		Casey (3)
adult	24	English	561		Casey (3)
adult	16	Polish	577		Casey (3)
adult	18	Dutch	635		Casey (3)
adult	18	Beveren	531		Casey (3)
adult	10	Chinchilla	548		Casey (3)
adult	13	Rex	595		Casey (3)
adult	8	French Silver	607		Casey (3)
adult	6	American Blue	713		Casey (3)
adult	4	New Zealand	798		Casey (3)
adult	3	Flemish	388		Casey (3)
adult	2	Gouda	462		Casey (3)
adult	1	Tan	690		Casey (3)
adult	4	—	170		Hueper (8)
Average			552.5		

for the adult rabbit, regardless of sex or strain, is 552,500. A seasonal increase from fall to spring of 35,300 platelets per cmm. of blood was found by Casey, Rosahn, Hu and Pearce (3) in male rabbits of different strains.

Hematocrit. The average of the reported values (Table V) for volume of packed red cells in adult rabbits, regardless of sex or strain, is 41.39 per cent. Female rabbits were shown by Wintrobe, Shumacker and Schmidt (12) to have a slightly higher hematocrit than male rabbits. A starvation period of 24 hours was found, by Cole, Allison, Murray, Boyden, Anderson and Leathem (4), to cause a 9.1 per cent. decrease in the volume of packed red cells.

TABLE V.
Hematocrit.

Age of Rabbits	Sex and No. of Rabbits	Strain	Hematocrit per cent.		Reported by
			Average	Range	
adult	26	—	42.2	35.7–48.1	Cole (4)
adult	4	—	46.0		Hueper (8)
adult	5	New Zealand white	41.85	37.2–46.6	Rapoport (9)
adult	4	albino	38.8		Sharpe (10)
adult	35 M	—	39.4		Wintrobe (12)
adult	26 F	—	40.1		Wintrobe (12)
Average			41.39		

Blood Volume. The blood volume of 20 normal rabbits was found, by Sanders, Florey and Barnes (18) to range from 105 to 120 cc. (4.3 to 5.0 per cent. of the body weight).

Sedimentation Rate. The one-hour sedimentation rate of the blood of normal rabbits was reported by Sharpe and Bisgard (10) to average 1.0 mm. and by Gregg (7) to range between 2.0 and 4.0 mm. Hueper (8) found no sedimentation at the end of one hour. A normal rabbit blood sedimentation rate (Linzenmeier technic) of 1,020 to 2,520 minutes was found by Rix (19).

Coagulation Time. An average coagulation time of 0.93 minute, with a range of 0.5 to 2.0 minutes, was found by Smith (11) using 24 normal adult rabbits. An average coagulation time of 8 minutes was found for 4 normal rabbits by Hueper (8).[3]

Specific Gravity. The average specific gravity of whole blood from normal rabbits was found, by Dougherty and White (6), to be 1.053.

Resistance of Red Blood Cells. No references to this were found in the literature.

The average values recorded here for the number and size of the red blood cells compare favorably with those found by Scarborough (20). The values for total platelets, coagulation time and blood volume, as well as for eosinophils and lymphocytes per 100 leukocytes, are somewhat lower than those recorded in Scarborough's review. Higher values than his were found for total leukocytes, and for pseudo-eosinophils and basophils per 100 leukocytes.

In addition, values are given for hematocrit and blood sedimentation rate.

(To be continued)

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[3] Both of these average values are outside the range, 1 to 6 minutes, reported by Scarborough (20), for 4 authors who had studied more than 57 animals.

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BOOK REVIEWS.

ENDLESS HORIZONS, by Vannevar Bush. 182 pages, 15 × 23 cms. Washington, D. C., Public Affairs Press, 1946. Price $2.50.

No one is better qualified than the widely known author of this book to summarize the advances in scientific knowledge and point the way to facilitate further development for the good of mankind. The book consists of a series of papers of wide coverage but yet with a singlness of purpose on the problems and opportunities confronting science. They are from recent writings, speeches, and reports brought together at the suggestion of the American Council of Public Affairs.

The basic philosophy and methods of approach of the author are unique and fascinating as can be seen in the very outset in a paper entitled "The Inscrutable Past." Here the reader reviews the technical accomplishments of another period—the early nineteen thirties—by considering for example a professor in some northern urban university. An attempt is made to appreciate his life by divesting ourselves of the scientific knowledge gained in only the short period since then. The items of this knowledge when recounted by the author seem incredible. The professor in his day seemed quite content. The recount however causes us today to lose some of our satisfaction and be better prepared for advance.

Later in a series of papers there is presented a fund of thought-provoking information, in giving and discussing recommendations requested by the President of the United States on making known to the world the contributions to scientific knowledge obtained from our war effort, the organizing of a program for future work in medicine and related sciences, the part of the government to aid research activities in public and private organizations, and a means of discovering and developing scientific talent in American youth. This truly is a program for tomorrow. It is stated, "Our population increased from 75 million to 130 million between 1900 and 1940. In some countries comparable increases have been accompanied by famine. In this country the increase has been accompanied by more abundant food supply, better living, more leisure, longer life, and better health." This trend can continue. But we must be ever vigilant in directing our course.

The latter part of the book contains an already well-known paper—one which every engineer should read—entitled "The Qualities of a Profession." This traces briefly the relationship of engineering to other professions, the professional traditions which engineers inherit, and the outlook for the engineering profession in view of its unique relationship to society. There has been much controversy of late with regard to our patent system. The author's viewpoint on the need of patent reforms as given in another paper should meet with wide interest.

R. H. OPPERMANN.

GALVANIZING HANDBOOK, by John R. Daesen. 166 pages, illustrations, 18 × 25 cms. New York, Reinhold Publishing Corporation, 1946. Price $5.25.

A review of statistics on tonnage, efficiency and coating weights for various classes of galvanized products brings to light a surprising inefficiency of the process of zinc coating iron and steel. Considerable variation exists when comparing sizes and shapes of products, and irregular operations. The operations reporting higher, recovery of zinc have by no means exhausted the possibilities of reduction of losses, but it must be recognized that such savings must be balanced with factors including production rate from a given capital investment.

This handbook presents the basic principles involved in the industry and demonstrates with good photographic examples the nature and cause of many defects. Arranged in the form of a text, it has twelve chapters which are subdivided into quite small portions, all of which are well indexed in a subject index in the back. The main subject headings include the Mechanism of Protective Action, Typical Structure of a Zinc Coating, Fluidity Control with Temperature

and with Aluminum Addition, Cleaning and Pickling, and Fluxing. The descriptive matter is brief and clear.

The work should make for better efficiency in galvanizing, resulting in reduced costs and better quality.

R. H. OPPERMANN.

QUALITY THROUGH STATISTICS, by A. S. Wharton. 60 pages, graphs, 14 × 22 cms. London, Philips Lamps Ltd., 1945. Price $1.50 (paper).

During the war period the production of a large number of serviceable articles has been more important than turning out a smaller number of beautifully finished products. Now competition is coming into the picture and orders will go to firms which produce a high class of goods. This booklet explains the usefulness of certain statistical methods in improving manufacturing quality. Written especially for the manufacturing plants of the British Isles, the work dwells on the sampling inspection ticket, probability tables, defective chart, multi-gage chart, height histogram, measurement graph, methods card, budgetary control, and grading graph. Emphasis is placed that only simple schemes which can be comprehended readily will prove satisfactory.

R. H. OPPERMANN.

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